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APPLICATION OF IMAGE THEORY TO MIXING IN STREAMS

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APPLICATION OF IMAGE THEORY TO MIXING IN STREAMS

A Thesis Presented

by

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ABSTRACT

A mathematical solution is proposed for the two-dimensional turbulent mixing and advection equation governing mass transport in open channel flow. The solution is obtained through the use of image theory. The solution is adaptable to a new, rapid, and inexpensive method for determining localized dispersion and diffusion coefficients for mathematical water pollution modeling. Local field tests of the method were conducted and the results indicate that typical stream dispersion coefficients are an order of magnitude smaller than those generally reported in the literature. The method presented will lead to more accurate physical transport modeling and will thus allow better evaluation of the chemical and biological processes occurring in natural streams.

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CHAPTER 1 INTRODUCTION

1.1. Justification

Within the past decade the problem of water pollution control has come to be recognized as a major technological and legislative challenge. The Water Quality Act of 1965 outlined what was thought to be a comprehensive plant of attack. At the State level, all major water bodies were to be classified as to use. Water quality standards associated with each use-class would then be adopted. Thus, appropriate wastewater treatment could be implemented at a level compatible with the established water quality goals. Unfortunately the progress to date has fallen far short of original expectations.

Costs associated with wastewater treatment are high. Federal (Environmental Protection Agency) and state grant-in-aid programs have been instituted to relieve local government of part of the capital costs of treatment plants and major intercepting facilities. Major collection facilities are also eligible for aid under other federal programs (e.g. Department of Housing and Urban Development, Farmers Home Administration). Operating costs associated with sewage treatment must be borne entirely by local governments.

A primary interest of the environmental engineering profession has been to develop and apply less costly techniques to accomplish a given degree of treatment. Since receiving stream standards are applicable rather than effluent standards, a major consideration arises as to what

degree of treatment is necessary in order to meet specific stream quality criteria. Specific answers are not readily obtained. Multiple local sources of wastewater complicate the matter further. While methods of systems analysis are well suited toward solution of such problems the methods pre-suppose the availability of reasonably accurate mathematical models describing interactions of the physical system. Such models have been difficult to formulate and, when applied, have been of marginal accuracy. Consequently a truly rational approach to water quality management has been retarded.

Many investigators have recently turned their attention to basic water quality model development. The success of their efforts to date is essentially unknown as simulation of the behavior of the same system from which experimental coefficients were determined does not truly verify such models. However, various existing physical systems have been simulated mathematically with substantial success.

1.2. Objective

The objective of this thesis is to present a refinement in the application of contemporary mathematical water quality simulation techniques. The proposed refinement concerns the physical mixing processes termed dispersion and diffusion. A method is formulated whereby field measurement of diffusion and dispersion coefficients in turbulent flow systems may be rapidly and economically obtained.

1.3. Background

If natural streams possessed no assimilative capacity the question as to what degree of treatment should be given would become technically, at least, quite trivial. This, however, is not the case. The essential

function of a mathematical model is to predict accurately the resulting stream water quality due to some specific network of artificial and/or natural waste discharges. The most desirable model is one in which both temporal and spatial distributions of water quality are predicted under unsteady flow conditions. The model itself is simply an equation or group of equations which relate a set of independent variables to a set of desired dependent variables. Thus any mathematical equation which describes the behavior of some physical, chemical, or biological phenomenon is a mathematical model. Newton's Laws of Motion, Boyle's Gas Laws, and the Biochemical Oxygen Demand (BOD) equation are all examples of mathematical models.

The complexity of any model varies directly with the complexity of the system represented. The model for a sphere falling in a vacuum is easily written. Solutions for the acceleration, velocity, and displacement of the sphere with time are readily obtained by differentiation or integration of the original model (equation). However, consider a non-symmetrical object falling in a heterogeneous atmosphere with variable winds prevailing. The model for this case would be quite difficult to formulate and particular solutions for velocity, acceleration, etc. as a function of time may not be analytically possible. Approximate solutions, however, might be obtained by making certain assumptions concerning the forces acting on the body. For example, one might linearize atmospheric density variations, determine an effective (resultant) one-directional wind force and, approximate the object shape by substituting an appropriate geometric form.

A natural stream or estuary represents an extremely complex, dynamic environment. When man's waste products are deposited into such a system, the complexity of the system's reactions increases even further. It is doubtful if man will ever devise a model to describe accurately all resulting reactions. On the other hand, the benefits to be derived from even the crudest mathematical simulations of system response are of great economic importance since such simulations will serve as a basis for rational water quality decision making.

The first successful attempt at stream modeling was made by E. B. Streeter and W. B. Phelps in 1926 (1). Using the Ohio River for their verification, the authors were able to approximate dissolved oxygen concentrations downstream from a waste source. The simplifying assumptions made by the authors were rough, to say the least, but nevertheless the model worked well enough to show that the system could be mathematically approximated.

The mathematical computations required in using the Streeter-Phelps formulas were rather tedious and in 1948, Thomas (2) worked out a nomographic method which greatly simplified the calculations. Thomas also presented a method of calculating the pollution-load capacity for a stream using the basic Streeter-Phelps equations. In 1958, Churchill (3) presented a totally different model of stream purification based entirely on statistical analysis. However, large amounts of data were needed in order to apply the method and the results were no better than those resulting from the Streeter-Phelps formulation.

In 1960, D. J. O'Connor (4) brought new life into the modeling field with his paper "Oxygen Balance of an Estuary." In this paper,

Dr. O'Connor brought together much of the previous individual work which had been done in refining the coefficient determinations of the Streeter-Phelps equations, but O'Connor's biggest contribution was his inclusion of the well known Fickian diffusion theory in his mathematical model. Mathematical models of river and estaurine waste assimilation have developed rapidly since that time. Steady and unsteady state models have been developed for one- and even two-dimensional flow. Both numerical and analytical techniques have been used in gaining solutions for the models.

The purpose of this paper is to present a refinement in the application of the modern mathematical models which have been proposed by the above authors. This refinement is concerned with the physical concept of mixing as employed in the contemporary models. In the text to follow, a rational method shall be given whereby field determination of mixing coefficients can be accomplished much more rapidly and economically than is presently possible. Further, the method should allow a much more accurate determination. To simplify the use of the method a general FORTRAN IV program is given for rapid analysis of the coefficients utilizing a minimum amount of field data. Finally the results of a test run of the method are presented.

CHAPTER 2 FUNDAMENTALS OF TURBULENT MIXING

2.1. Mixing Zones

When a conservative material, dissolved or suspended in a liquid is discharged continuously in a steady turbulent stream, a steady state concentration profile will develop downstream. Such a system may be divided up into three mixing zones. Figure 1 depicts the steady state condition which will appear. In the zone immediately adjacent to the outfall, a three-dimensional concentration profile will appear at every point. This is, of course, due to incomplete mixing in the lateral, longitudinal and vertical directions. This zone is designated as the three-dimensional mixing zone. The lower boundary of this zone will be marked by the point at which all vertical concentration profiles vanish. Downstream of this boundary will be located the two-dimensional mixing zone where concentration profiles exist only in the lateral and longitudinal directions. Vertical gradients are the first to disappear in a natural stream as the depth of the stream is generally an order of magnitude less than the width. However, eventually the lateral gradients also will vanish and this point then marks the lower boundary of the two-dimensional mixing zone as well as the upper bound of the one-dimensional mixing zone. The one dimensional mixing zone extends indefinitely. This zone is by far the largest zone and justly receives the greatest amount of mathematical treatment. A concentration gradient exists only in the longitudinal direction. In describing, mathematically, the concentration profile in this last zone, a one-dimensional model (equation) will be appropriate. This situation simplifies the mathematical model.

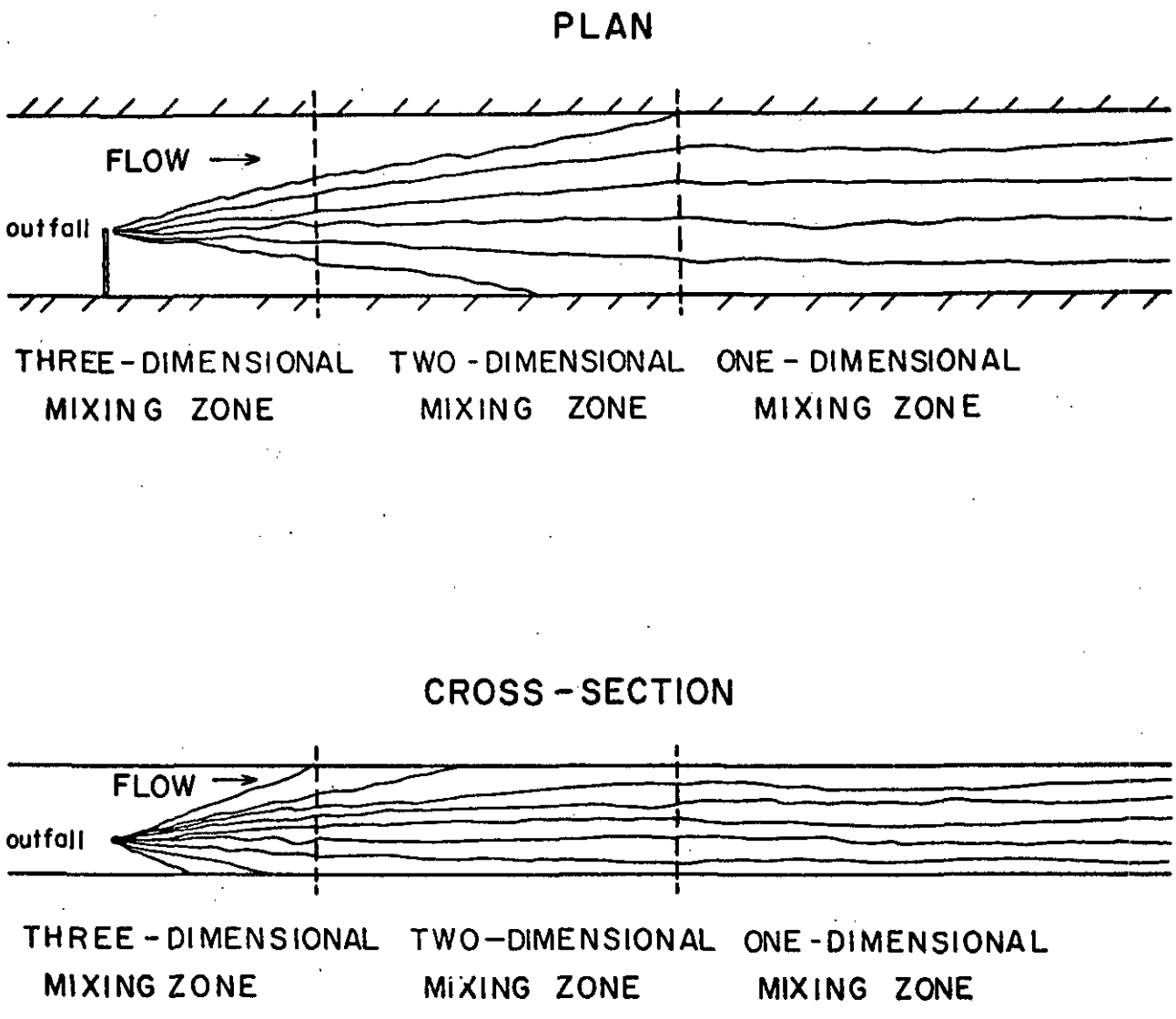


Figure 1. Turbulent Mixing Zones.

2.2. Transport Mechanisms

Only three physical mechanisms are at work causing non-reactive materials to become transported in a stream: molecular diffusion, advection and turbulent mixing.

Molecular diffusion is an extremely slow process which is due to Brownian (random molecular) motion. In 1855 Fick (5) proposed the theoretical model of molecular diffusion to be

$$\frac{\partial M}{\partial t} = -D_m A \frac{\partial C}{\partial 5} \qquad (1)$$

His equation states that the time rate of mass transfer, $\frac{\partial M}{\partial t}$ through the surface A, is proportional to the concentration gradient $\frac{\partial C}{\partial s}$ across that surface. The constant of proportionality D_m was termed the molecular diffusion coefficient*. The direction of mass transfer is from a region of higher concentration to one of lower concentration. Now since a region of initially high concentration will suffer a decrease of mass with time, the rate of mass change is logically considered to be negative. Thus a minus sign is applied to the right side of the equation allowing the coefficient to take on a positive value. As an example of molecular motion, consider the injection of a drop of dye into a beaker of visually quiescent water. With the progression of time, the natural Brownian motion of the molecules will cause the dye to diffuse in all directions. Eventually, the dye would become completely mixed uniformly throughout the entire container. Once mixed this uniform concentration pattern would remain indefinitely. Molecular diffusion, however, is not a rapid

*Molecular diffusion is considered to be a completely random phenomonon.

phenomona and its significance in mixing and transport of materials in natural streams is so minor that it can safely be neglected (6,7).

Advection is the name normally given to the longitudinal mass transport occurring as a result of the hydraulic driving force. Transport by advection only is commonly referred to as plug flow or flow without mixing. Mathematically, advective mass transport is written as

$$\frac{\partial C}{\partial t} = -U \frac{\partial C}{\partial X} \qquad (2)$$

where U, the average cross-sectional velocity is the driving force. Occasionally the term "convection" will be used synonomously with advection. In view of the directional implication of the prior term, such usage is deemed inappropriate.

Turbulent mixing is the mechanism by which materials are transported in all directions as a result of the natural velocity gradient forces existing in turbulent flow. In a turbulent stream, unsteady velocity gradients are evident in the lateral, vertical and longitudinal directions as a result of the hydraulic properties of the stream combined with the viscous properties of the fluid. There has been a great deal of confusion in the literature regarding turbulent mixing terminology. Recently Edward Holley (8) attempted to clarify this matter by publishing a paper solely devoted to defining the term "diffusion" and "dispersion". Holley's descriptions are accepted by this writer and will be used throughout this paper. In order to characterize these terms, certain parameters must first be identified.

The cross-sectional model of stream flow shown in Figure 2 assumes a stream of infinite width so that there is no variation of any parameter in the direction perpendicular to the longitudinal plane. A slug of tracer is injected at time t_o. While moving downstream the tracer spreads vertically and longitudinally causing a decrease in concentration at any point within the cloud. The flow has a time averaged horizontal velocity $\bar{u}$, distributed vertically as shown in Figure 2. This $\bar{u}$ distribution is the velocity profile which could be measured by a pitot tube or a current meter. The instantaneous velocity u at any point might be distributed in time very irregularly as shown in Figure 3 for the condition on line AB. Thus another velocity distribution u' can be defined as the difference between $\bar{u}$ and u at every point. The distribution is termed the instantaneous fluctuation from the time average velocity. One may then write that

$$u = \bar{u} + u'$$

The average value of $\bar{u}$ over the vertical depth is defined to be U. Now the velocity distribution defined by the difference between the cross-sectional average, u, and the time average, $\bar{u}$, is designated as u". One may write then that

$$\bar{u} = U + u''$$

In an analogous manner, the concentration distributions depicted in Figure 3 may be defined. The instantaneous concentration at any point is c. The time average concentration at a point is $\bar{c}$, where this is normally the value determined by analysis of a grab sample. If it could be measured, c' would be the instantaneous fluctuation from the time average $\bar{c}$.

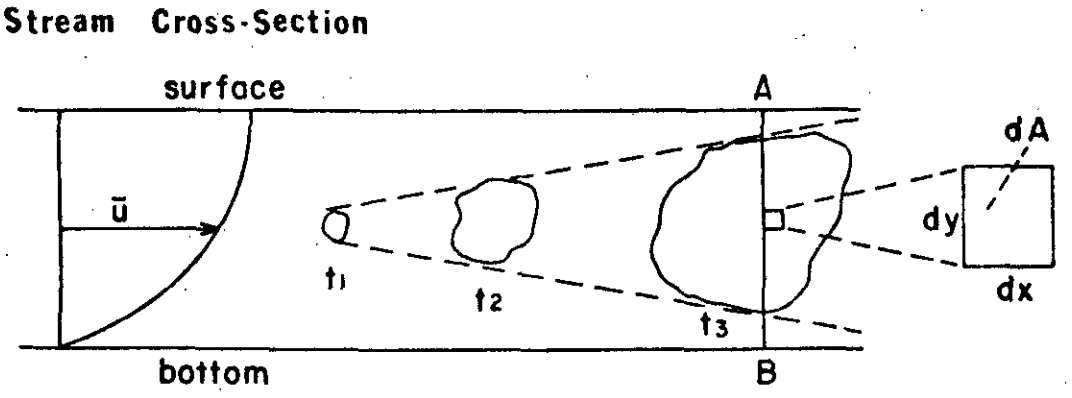


Figure 2. Two-dimensional Transport (after Holley (8)).

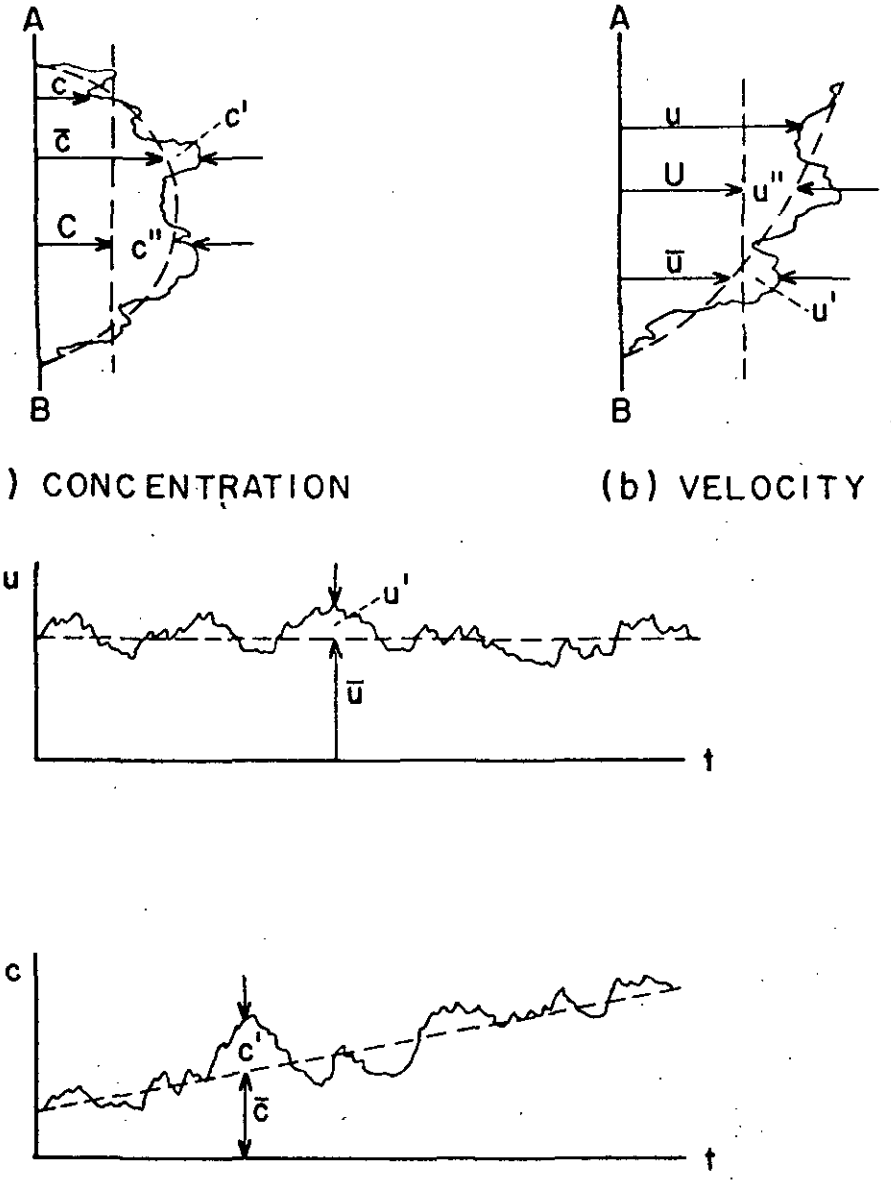


Figure 3. Parameters of Turbulent Transport (after Holley (8)).

Thus

$$c = \bar{c} + c'$$

With these definitions in mind, the terms diffusion and dispersion may be discussed. Both processes concern the spreading of materials which are dissolved or suspended in a fluid. The term "diffusion" implies motions which are entirely random. Diffusive movement occurs as a result of random path selection by finite fluid cells in a velocity field. Dispersion is considered to be a more orderly mixing process. Dispersive movement occurs as a direct result of the actual character of the velocity field. The process of advection is thus strongly related to the process of dispersion. Thus cells randomly select a velocity path (diffusion) but are transported at a velocity consistent with the path selected (dispersion). Following a finite advection period the cell randomly selects a new path (diffusion).

In one-dimension the processes of mixing following a point source outfall may be analogous to the "drunkard's walk" whereby the late night closing of a tavern causes all the intoxicated occupants to be removed outside to the street. After each interval of time, each "drunk" moves one step length but whether the step be forward or backward is entirely random. Now regularly scheduled bus service is available at every street corner in the area and each "drunk" who happens to be at the right corner and at the right time may be collected by the bus and removed to another location. The drunks get on and off each bus randomly, but since the buses operate on fixed schedules the final distribution of drunks depends strongly on the bus schedule. The bus schedule is analogous to the time

average velocity distribution since both are fixed. The final distribution of "drunks" or cells resulting from this non-random transport is referred to as dispersion. Dispersion is caused by "busing" of the particles, that is advection at the different velocities of different stream lines. The diffusive process causes the cells particles to "get on and off the bus".

The primary mechanism for spreading (mixing) in a stream is the time-average spatial distribution of velocity, $\bar{u}$. Of lesser importance is the mechanism of turbulent diffusion caused by the random temporal velocity distribution, u'. These spatial velocity variations are far more effective at spreading out particles than are the temporal variations. Consequently, the term "dispersion" is applied to the spreading out due to spatial velocity differences, and the term "diffusion" is applied to the spreading by random temporal velocity fluctuations.

The first important experimental study of dispersion in turbulent flow was published by G. I. Taylor (9) in 1954. He approached turbulent mixing in a manner directly analogous to Fickian molecular diffusion. Taylor experimented with turbulent pipe flow and thus spatial velocity variations were negligible. This greatly reduced the dispersion effect and relegated the problem to that of one-dimensional transport. The one-dimensional diffusion-advection model was written as,

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} = D \frac{\partial^2 C}{\partial X^2} \qquad (3)$$

where D was designated as the coefficient of turbulent diffusion. The coefficient D is thus a composite of the hydraulic and viscous properties

of the particular flow system and is not intended to be directly measurable. From the discussion just presented, it is obvious that Taylor's pipe flow model is not directly applicable to open channel flow. Many workers have nevertheless directly applied Taylor's model to one-dimensional open channel flow. However, a rather rigorous mathematical analysis of the diffusion-dispersion-advection process yields an equation, which when applied solely to the one-dimensional mixing zone, reduces to precisely the same mathematical equation,

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} = E\,\frac{\partial^2 C}{\partial X^2} \qquad (4)$$

In this equation, E is designated as the coefficient of longitudinal dispersion. Turbulent diffusion is included in the dispersion term. Workers who have supported and utilized Taylor's pipe diffusion model for one-dimensional stream analysis lack in terminology only. Mathematically, the Talor equation is generally acceptable for one-dimensional mixing, but the mixing coefficient should be referred to as a longitudinal dispersion coefficient, not a longitudinal diffusion coefficient. Oceanographic and atmospheric turbulent

diffusion is also generally treated on the basis of this model (6, 10).

2.3. Advective-Mixing Equation

The basic equation used by Taylor (9) will now be developed from a one-dimensional mass balance analysis. It should be kept in mind that such an analysis is applicable only within the one-dimensional mixing zone beginning some finite distance below the outfall point. Figure 4 depicts a reach of length ΔX of a rectangular channel within the one-dimensional mixing zone. The cross-sectional area A is considered constant for the reach. The instantaneous concentration at the center of the reach is c. The instantaneous velocity is in the positive X direction. The net mass transport through the center of the reach is considered to be the sum of the advective transport, uAc, plus the diffusive mass transport as defined by the Fickian diffusion approach, $-DA\frac{\partial C}{\partial X}$. Thus

$$\text{Transport thru center} = uAC - DA\frac{\partial C}{\partial X}$$

The mass transport into the reach can then be approximated by writing a Taylor series expansion at the $-\frac{\Delta X}{2}$ location and truncating the series after the first order term.

$$\text{Advective mass inflow} = uAC + \frac{\partial}{\partial X}(uAC)\left(-\frac{\Delta X}{2}\right)$$

$$\text{Diffusive mass inflow} = -DA\frac{\partial C}{\partial X} + \frac{\partial}{\partial X}\left(-DA\frac{\partial C}{\partial X}\right)\left(-\frac{\Delta X}{2}\right)$$

$$\text{Total mass inflow} = uAC + \frac{\partial}{\partial X}(uAC)\left(-\frac{\Delta X}{2}\right) - DA\frac{\partial C}{\partial X} + \frac{\partial}{\partial X}\left(-DA\frac{\partial C}{\partial X}\right)\left(-\frac{\Delta X}{2}\right)$$

The mass transport out of the reach is formulated similarly at the downstream boundary:

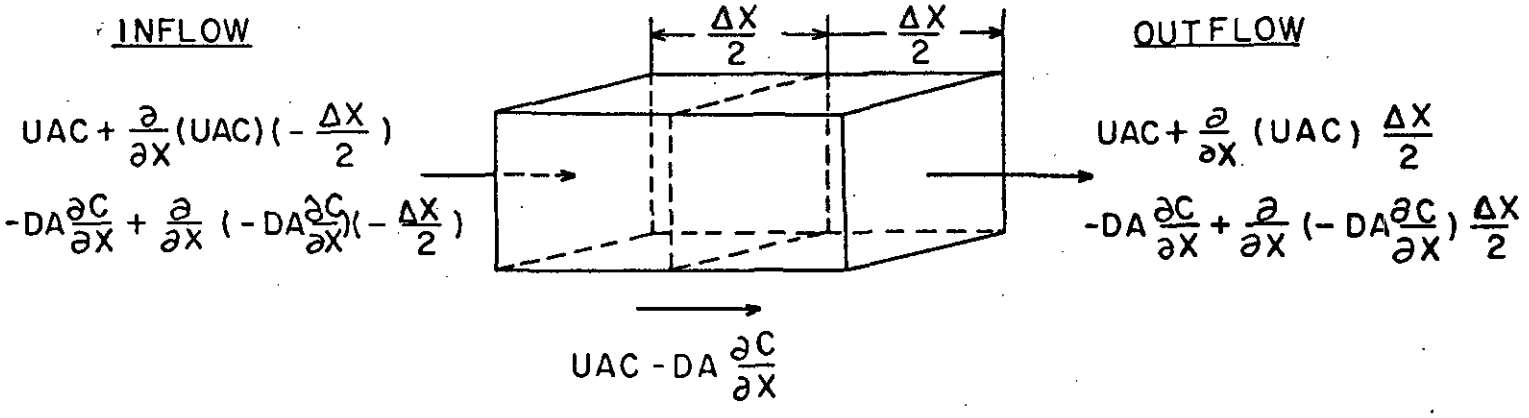


Figure 4. Flux Rates Through An Elemental Reach of A Rectangular Stream Channel.

Advective mass outflow = $uAC + \frac{\partial}{\partial X}(uAC)\left(\frac{\Delta X}{2}\right)$

Diffusive mass outflow = $-DA\frac{\partial C}{\partial X} + \frac{\partial}{\partial X}\left(-DA\frac{\partial C}{\partial X}\right)\left(\frac{\Delta X}{2}\right)$

Total mass outflow = $uAC + \frac{\partial}{\partial X}(uAC)\left(\frac{\Delta X}{2}\right) - DA\frac{\partial C}{\partial X} + \frac{\partial C}{\partial X}\left(-DA\frac{\partial C}{\partial X}\right)\left(\frac{\Delta X}{2}\right)$

Now a mass balance may be written for the reach utilizing the Law of Mass Conservation; that is, the rate of accumulation of mass is equal to the rate of mass inflow less the rate of mass outflow. The rate of mass change may be written as $\frac{\partial}{\partial t}(A\Delta XC)$ since the mass within the reach is $A\Delta XC$. Thus,

$$\frac{\partial}{\partial t}(A\Delta XC) = -\frac{\partial}{\partial X}(uAC)\,\Delta X + \frac{\partial}{\partial X}\left(DA\frac{\partial C}{\partial X}\right)\Delta X \qquad (5)$$

Now for a small elemental reach the area A may be considered constant. Dividing through by $A\Delta X$ gives,

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial X}(uC) + \frac{\partial}{\partial X}\left(D\frac{\partial C}{\partial X}\right) \qquad (6)$$

and for u= constant over the elemental reach then,

$$-\frac{\partial}{\partial X}(uC) = -u\frac{\partial C}{\partial X}$$

and thus,

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial X} + \frac{\partial}{\partial X}\left(D\frac{\partial C}{\partial X}\right) \qquad (7)$$

Finally for D=constant over the reach, the above reduces to

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} = D\frac{\partial^2 C}{\partial X^2} \qquad (8)$$

Equation 8 serves as a basis from which many contemporary models have been derived. It has already been pointed out however that the above analysis is incorrect since it is founded on the notion that longitudinal mixing is entirely random. The more accurate and rigorous analysis, however resolves into the same equation, although the coefficient D must be referred to as a dispersion coefficient. The refined analysis will be presented in the next section of this paper. The result, for one-dimensional mixing, will nevertheless be the same as the above basic Taylor equation.

An equation governing two-dimensional diffusive advection is readily obtained as above by simply applying the same Taylor mass balance approach for one more transport direction.

$$\frac{\partial C}{\partial t} + u\,\frac{\partial C}{\partial X} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial X}\left(D_x\,\frac{\partial C}{\partial X}\right) + \frac{\partial}{\partial y}\left(D_y\,\frac{\partial C}{\partial y}\right)$$

Here, D_x and D_y refer to turbulent diffusion coefficients acting in the longitudinal and lateral directions. By assuming the reach is relatively small the diffusion coefficients may be considered constant, thus

$$\frac{\partial C}{\partial t} + u\,\frac{\partial C}{\partial X} + v\,\frac{\partial C}{\partial y} = D_x\,\frac{\partial^2 C}{\partial X^2} + D_y\,\frac{\partial^2 C}{\partial y^2} \qquad (9)$$

One might now assume that D_x and D_y are equal and the equation will simplify to

$$\frac{\partial C}{\partial t} + u\,\frac{\partial C}{\partial X} + v\,\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial y^2}\right) \qquad (10)$$

However, Equations 9 and 10 are not the appropriate equations for stream-flow analysis.

Equation 9 was derived from the Taylor turbulent diffusion analysis. Instead of considering turbulent diffusion, the Taylor derivation should have dealt only with molecular diffusion, D_m since u, v, and c represent instantaneous values and thus by definition must incorporate turbulent diffusion and dispersion. Since molecular diffusion is considered constant in any direction, the correct result of the Taylor analysis of two dimensional flow would be

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial X} + v \frac{\partial C}{\partial y} = D_m \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial y^2}\right) \qquad (10)$$

Now this equation would be completely correct for two-dimensional open channel flow as well as pipe flow. The equation represents the movement of suspended or dissolved material in a two-dimensional flow field. It is emphasized, however, that the instantaneous values of u and v must be used in the equation. Also any solution to the problem should yield instantaneous values of c. The terms u and v cannot ordinarily be measured and instantaneous values of c are of little value. What is measurable are the values of $\bar{u}$ and $\bar{c}$. Furthermore, the value of $\bar{v}$ is zero for essentially straight channel flow since there is no net flow through the stream boundaries. Now the following substitutions may be made in Equation 10.

$$u = \bar{u} + u'$$

$$C = \bar{c} + c'$$

$$v = v'$$

Now averaging the modified Equation 10 with time gives

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial X} = D_m \frac{\partial^2 \bar{c}}{\partial X^2} + D_m \frac{\partial^2 \bar{c}}{\partial y^2} + \frac{\partial}{\partial X}(-\overline{u'c'}) + \frac{\partial}{\partial y}(-\overline{v'c'}) \qquad (11)$$

where the bars indicate time averaged values. The last two terms of Equation 11 include advection associated with the u' and v' fluctuations. It has been confirmed experimentally that for many situations this advective transport follows a diffusive process analogous to Fick's First Law, that is, that the transport associated with turbulent fluctuations is proportional to the gradient of $\bar{c}$ (11). This is not difficult to visualize since these fluctuations would be distributed randomly. Thus the following equations may be introduced to represent the fluctuating turbulent advection.

$$\overline{u'c'} = -D_x \frac{\partial \bar{c}}{\partial X}$$

$$\overline{v'c'} = -D_y \frac{\partial \bar{c}}{\partial X}$$

Again D_x and D_y would be designated as longitudinal and lateral diffusion coefficients. Now Equation 10 may be rewritten as

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial X} = (D_m + D_x)\frac{\partial^2 \bar{c}}{\partial X^2} + (D_m + D_y)\frac{\partial^2 \bar{c}}{\partial y^2} \qquad (12)$$

Equation 12 is the equation for two-dimensional mass transport which includes both turbulent and molecular diffusion. The equation applies to both one- and two-dimensional mixing zones, since the one-dimensional flow is a special case of two-dimensional flow.

In applying this equation to the one-dimensional mixing zone the following substitutions are made

$$\bar{u} = U + u''$$

$$\bar{c} = C + c''$$

where U and C are the average values of velocity and concentration in a cross-section. The term $\bar{u}$ cannot be replaced by U in Equation 12 since such a substitution would not account for the advection due to the velocity variation u". Now by integrating or averaging Equation 12 with respect to time along with the above substitutions, the equation may be rewritten as

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} = (D_m + D_x) \frac{\partial^2 C}{\partial X^2} + \frac{\partial}{\partial X} (-\overline{\overline{u'' c''}}) \qquad (13)$$

The double bar indicates the average value for the cross section. Now the transport associated with u" is proportional to the longitudinal gradient (7) so that

$$E \frac{\partial C}{\partial X} = -\overline{\overline{u'' c''}} + (D_m + D_x) \frac{\partial C}{\partial X} \quad \text{for } c'' << C \qquad (14)$$

The transport coefficient associated with the longitudinal gradient of c in Equation 14 is called the coefficient of longitudinal dispersion. Thus the longitudinal molecular and turbulent diffusion are combined with the convective transport due to lateral variations in velocity and concentration to make up the dispersion coefficient, E. Now in uniform turbulent flow, both molecular and turbulent diffusion contribute only about 1% or less of the total dispersion (12). Thus the last term of

Equation 14 may be neglected. Combining Equations 13 and 14 gives

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} = E \frac{\partial^2 C}{\partial X^2} \qquad (15)$$

This result is precisely that obtained by Taylor except for the terminology. Again this equation applies only to the one-dimensional mixing zone. The equation is not valid near a point outfall, but becomes applicable only when vertical and lateral concentration gradients become negligible downstream from the outfall. In this one-dimensional mixing zone the terms diffusion and dispersion are often used interchangeably; however, the problem resolves itself since the mathematical model is the same in either case.

2.4. Methods

There are numerous methods presently available for determining the value of the longitudinal dispersion coefficient. The methods may be classified as either empirical or analytical.

2.4.1. Empirical Approach

In 1954 G. I. Taylor presented the results of a careful laboratory study of the turbulent diffusion process in pipe flow (9). Taylor came up with the following empirical relationship

$$D = 10.1\, au^*$$

Where, a = Pipe Radius (U)

u^* = Shear Velocity = $\sqrt{\tau_o/\rho}$ (L/T)

γ_o = Wall Shear (L^2/T^2)

Elder, in 1959 (13) then adopted this approach to two-dimensional flow in an infinitely wide open channel. He found

$$D = 5.9\ du^*$$

Where d = Depth of Flow

Elder obtained his experimental results using a 1 cm deep water table of large width. Experimental results in natural streams, however have not substantiated these laboratory results. In fact dispersion coefficients for natural streams have varied from 50 to 700 ru^* (r = hydraulic radius) according to Fischer (7). The empirical method is, nevertheless occasionally used under certain conditions.

Harleman (14), has modified the Taylor equation for use in the upstream tidal region of streams.

$$D = 77\ rU_{xt}R_H^{5/6}$$

Where U_{xt} = Tidal velocity

R_H = Hydraulic Radius

r = Manning Roughness

The supposition here is that E depends primarily on the magnitude of the oscillating tidal velocity. In substituting this expression for E into the one-dimensional mixing equation (Equation 15), an analytic solution for the model will no longer be possible. Thus the empirical approach to the diffusion coefficient is rarely used.

2.4.2. Analytical Approach

The second and most popular approach to determining the dispersion coefficient is obtained by employing analytic solutions to the steady and unsteady one-dimensional models.

In a non-tidal stream the unsteady one-dimensional dispersive-advective equation (Equation 15) is readily solved for the following conditions. If at time t = 0 a quantity of mass M is injected at x = 0 while at all other points c = 0 then operational methods will give the solution (15) where A is the cross sectional area.

$$c(x,t) = \frac{M}{2A\sqrt{\pi Et}} \exp\left[-\frac{(x-Ut)^2}{4Et}\right] \qquad (16)$$

This solution is transcendental in E but can be solved for E by noting that the relative maximum concentration will occur when the argument of the exponential is zero, that is

for $x-Ut = 0$

$$C = C_{max} = \frac{M}{2A\sqrt{\pi Et}}$$

so that $E = -\frac{(x-Ut)^2/4t}{\ln (C_{max}/C)}$

Thus to find E, a known mass of tracer is injected and a spatial concentration profile is measured for any time t. The curve resulting should be normally distributed. The dispersion coefficient may then be calculated from the slope of $\frac{(x-Ut)^2}{4t}$ versus $\ln (\frac{C_{max}}{C})$. Due to the difficulty of measuring a simultaneous spatial concentration profile, several methods have been proposed to extract a value for E from the more readily obtainable time-concentration profile. It should be noted here however that the time concentration curve is not normal but is skewed as the denominator changes with time. Thackston, Hays and Krenkel (16) have performed an extensive analysis of all the popular methods of calculating E in streams from time concentration profiles. Their conclusions were that all methods

are unreliable at best, and highly inaccurate at worst. The authors also concluded that acceptable accuracy can only be provided through a least squares minimization analysis of the observed and theoretical concentrations. Such procedures are incorporated into the proposed method of determining E to be presented in this paper.

The ordinary method for determining the dispersion coefficient in an estuarine condition is to utilize the natural salinity gradient in conjunction with the steady state solution to the one-dimensional dispersive-advective equation. The steady state one-dimensional mixing form of Equation 15 is

$$U \frac{\partial C}{\partial X} = E \frac{\partial^2 C}{\partial X^2} \quad \text{or} \quad U \frac{dC}{dX} = E \frac{d^2 C}{dX^2}$$

The solution for this equation for the boundary condition $C = C_o$ (the constant salinity of the ocean) at $x = 0$ is

$$C(x) = C_o \exp\left(-\frac{UX}{2E}\right)$$

This equation is then solved for E by

$$E = -\frac{Ux/2}{\ln (C/C_o)}$$

Although a steady state salinity intrusion is not physically possible due to the oscillating tidal effect, a pseudo-steady state may be established by interpreting the differential time as one discrete tidal cycle. If the instantaneous time utilized is either high water slack (HWS) or low water slack (LWS) then the net velocity within the estuary will be a result only of the fresh water discharge. Thus the effect of tidal velocity is excluded and the calculation of U is greatly simplified.

Now to determine the turbulent diffusion coefficient a longitudinal salinity profile is determined at either HWS or LWS. The value of E is then calculated as the slope of UX/2 versus in C/C_0 plot (Figure 5).

2.5. Limitations of Present Techniques

For the methods previously discussed certain critical limitations arise primarily from the assumption of one-dimensional mixing. For non-tidal streams the assumption of one-dimensional mixing can be theoretically justified by restricting the mathematical model to a reach beginning a good deal downstream of the outfall point. The upstream boundary of the reach (the one-dimensional mixing zone as described in Section 2.1) is defined as that point where lateral and vertical concentration gradients are no longer present. In applying the methods for calculating E using both the empirical and the theoretical approach, it is mandatory that the upstream boundary be located both in space and time. Employing a one-dimensional mixing model to a two- or three-dimensional mixing zone is by definition inappropriate. The corollary however is not true as the one-dimensional and two-dimensional models are special cases of the three-dimensional model.

Locating the upstream boundary of the one-dimensional model is a tedious problem. It may be located by observing the downstream concentrations of a continuous discharge of a tracer material. Lateral traverses with a continuous recording instrument will facilitate locating the one-dimensional mixing zone boundary. Time of travel to that boundary location must also be determined. An instantaneous tracer drop from the outfall is well suited

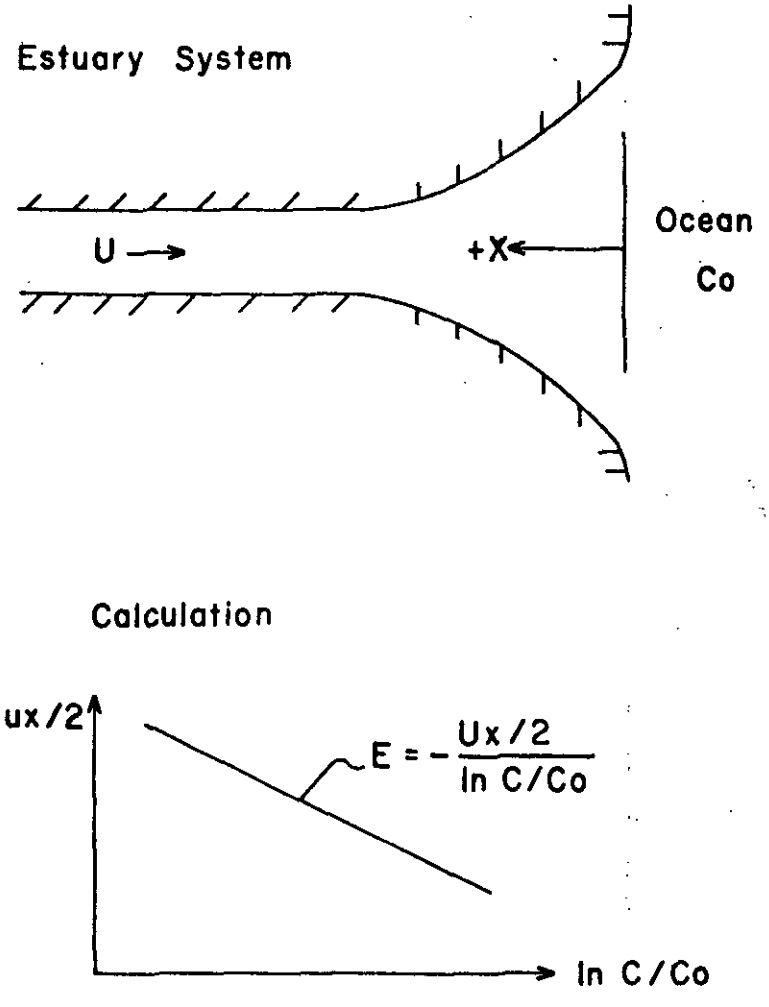


Figure 5. Determination of E by the Steady-state Salinity Profile of an Estuary.

for this purpose once the special location of the boundary has been identified. It should also be noted that discharge is a prime factor in determining the distance and time of travel to the one-dimensional mixing zone. As the discharge varies, so also does the boundary.

The only apparent way to get around this difficulty of locating the upstream boundary of the one-dimensional mixing zone is to introduce the tracer instantaneously and uniformly in the cross-sectional plane, thus precluding any need for lateral and vertical mixing. If the stream is relatively shallow, a single traverse while releasing the tracer at a uniform rate will accomplish the objective. This procedure is often used in field studies. If not entirely accurate, the method is at least far superior to estimating the one-dimensional boundary for a point release. The remaining alternative of neglecting the existence of the three- and two-dimensional mixing zones is considered most unacceptable by this writer except in small, highly turbulent streams.

Another item of considerable significance is in order for discussion at this point. Once the mixing distance for a particular outfall has been established, then the appropriate stream reach should be deleted in the actual mathematical water quality model. Furthermore, since most stream models involve rather large drainage areas and therefore several liquid waste outfalls, each outfall will require an appropriate mixing distance before the one-dimensional model can be applied. Thus in areas where multiple domestic and industrial waste outfalls discharge to a major stream the one-dimensional model appears most inadequate. This problem is overcome by treating each outfall and constructing a one-dimensional

model by superposition. The final concentration at some point downstream of, say four, outfalls will be equal to the sum of the concentrations due to each of the four individual outfalls, but only in the reach where complete vertical and lateral mixing has occurred for each discharge. Thus, the final model will in effect be the result of the appropriate superposition of four individual independent models.

Models of estaurine systems pose additional problems. The major drawback to the salt-balance method of calculating dispersion coefficients is the inaccuracy of the assumption of one-dimensional mixing. The most obvious departures from this assumption are the density currents caused by temperature and salinity differences in ocean and stream waters. These effects lead to vertical salinity gradients which limit vertical mixing. Also differential heating of the waters in the littoral zone (especially if tidal flats exist) cause lateral advective and diffusive movement. With the present methods available there seems to be little hope of overcoming the inadequacies of the one-dimensional assumption. Even if the pseudo-steady state method were to give a reasonable but crude estimate of longitudinal dispersion, actual application of the model is quite difficult, especially when multiple outfalls are present.

In the pseudo-steady state approach, the time increment is chosen to extend from one complete tidal cycle to another, selecting the discrete time as instantaneous high of low water slack. Then, just as in the case for streams, the mixing distance from any given outfall may be determined experimentally by the continuous addition of a tracer. The major shortcoming of these procedures is that a continuous model of the system will

not be available so that any diurnal fluctuations cannot be calculated. Also a rather large amount of tracer material and many chemical analyses are required.

A variation of the pseudo-steady state type model is to simulate a continuous one-dimensional mixing model by substituting a sinusoidal expression to simulate the oscillating tidal velocity. This is commonly done. However such a continuous model would be continuous only in the sense of time but not longitudinally as the one-dimensional mixing equation would not apply immediately up or downstream of existing waste outfalls. By superposition of an independent input and mixing distance for each outfall, sections of the model may be simulated, however the model will not provide continuous spatial predictions.

2.6. Significance of Dispersion

In the one-dimensional dispersive-advective model, the significance of dispersion varies greatly. Many researchers will neglect the dispersion process altogether for stream models in view of the greater importance of advection. In contrast, the process of advection may often be neglected in estuary models in view of the greater significance of dispersion. The basis for this will now be discussed.

A fundamental difference from a mechanics point of view between a stream and an estuary, is characteristic velocity. The average sectional velocity of a stream is normally higher and far more constant, in time and space, than that for estuaries. The net or average advective velocity of an estuary is typically at least an order of magnitude smaller than that of its influent stream. As an example, a characteristic velocity for streams may be 20 mi/day while for estuaries a velocity of 1 mi/day

(average advective velocity over a 24-hour period) might apply. Steady state spatial concentration profiles for one-dimensional mixing and advection of wastes below an estuarine and a stream outfall are shown in Figure 6 for various values of E. The effect of E for the typical stream situation is quite obviously of limited significance. By contrast, the relative effect of E for the estuary case is significant. Both curves can be readily developed for particular values by applying Equation 16.

The results of a similar analysis for the unsteady case are illustrated by Figure 7, a temporal concentration plot one mile downstream of the outfall. Particular values may be developed by applying Equation 15.

In considering the significance of dispersion in natural streams and estuaries one must also consider the relative values of the parameter being modeled. Time-concentration curves for a tracer mass release may vary over several orders of magnitude, and so a variation in the dispersion coefficient will have a substantial effect on the resulting curve. On the other hand, if the parameter to be modeled is dissolved oxygen then the possible range of values is considerably reduced. Dissolved oxygen values will normally be in the zero to ten parts per million (ppm) range with about 1 to 3 percent accuracy. Thus when dissolved oxygen is the parameter to be modeled, the degree of accuracy needed for the dispersion coefficient will be minimal.

Holley (8) has suggested that the relative importance of dispersion may be investigated for a given situation by forming a

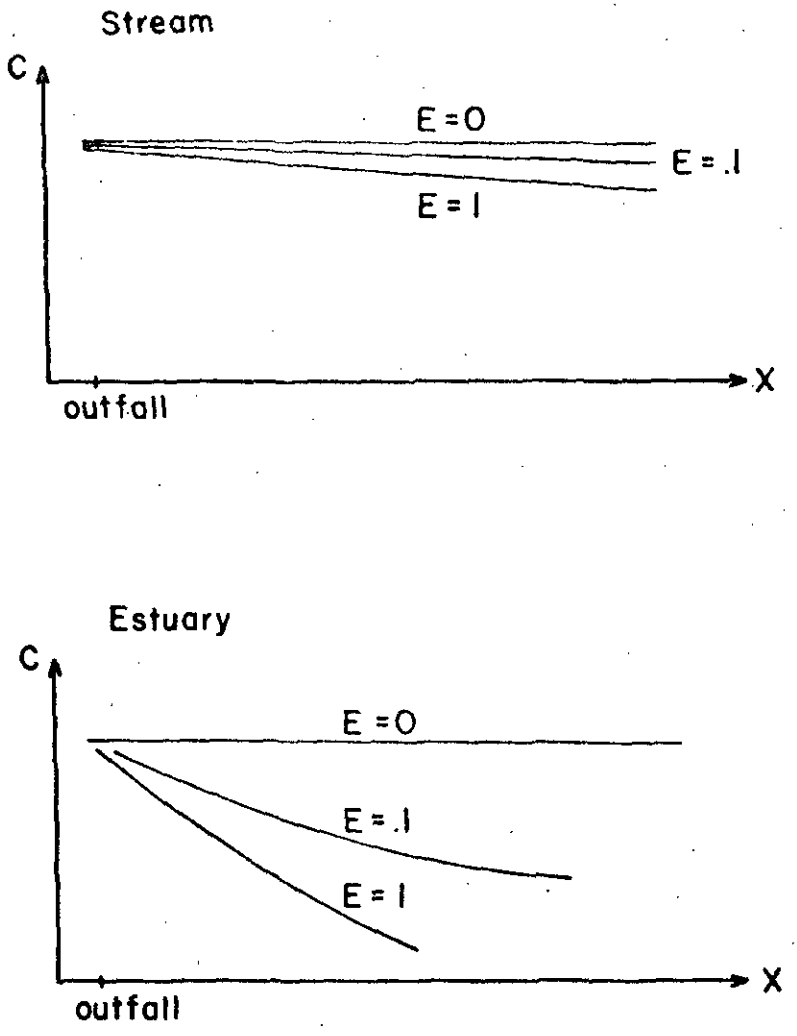


Figure 6. Steady-state Spatial Concentration Curves Indicating Significance of E in Streams and Estuaries.

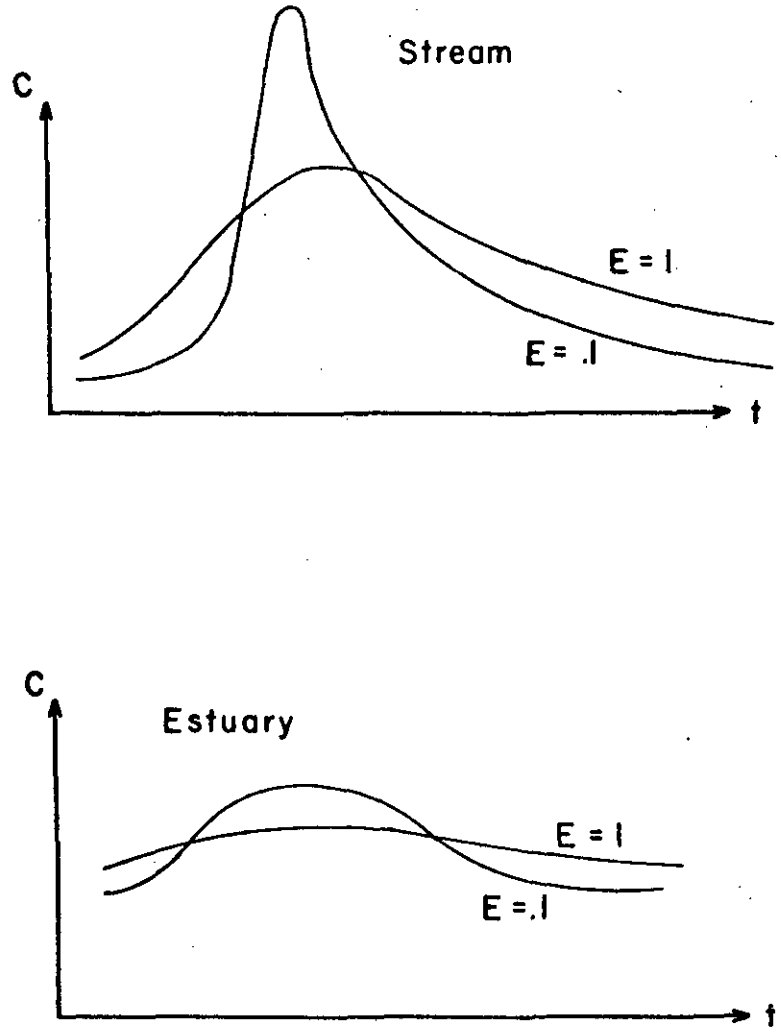


Figure 7. Unsteady Time-Concentration Curves Indicating Significance of E in Streams and Estuaries.

dimensionless term

$$H = \frac{\text{dispersive rate}}{\text{advective rate}} = \frac{E\,\frac{\partial C}{\partial X}}{UC} = \frac{E}{U}\,\frac{1}{C}\,\frac{\partial C}{\partial X} = \frac{E}{U}\,\frac{\partial(\ln C)}{\partial X}$$

Since H is defined as the ratio of dispersion rate to advection rate, a value of H less than unity indicates that the tracer is transported more slowly by dispersion than by advection. Thus for very small values of H the dispersive transport may be neglected. For a particular uniform flow, E and U will be constant, thus the value of H will depend on the steepness of the concentration gradient (more precisely the steepness of the gradient of the natural logarithm of the concentration). Since the oxygen concentration in an oxygen sag curve will change by only a few parts per million over distances of several miles, the value of H would be quite small. A BOD curve however might vary by one order of magnitude or possibly two in the same reach. The value of H would then be larger indicating that dispersion is a significant factor in the transport of this material.

A critical value for H, called H_c, would be chosen according to the degree of accuracy required. If it is sufficient to neglect factors contributing less than 1% of the transport, then let $H_c = 0.01$. Thus, the dispersion process may be dropped from the analysis if $H_c < .01$. One should remember here, however, that the effect of dispersion is cumulative so that no matter how small H becomes there may still be a significant dispersive effect very far downstream.

2.7. Recent Results

Three of the most extensively studied basins in this country in regard to pollution modeling have been the Delaware River, the Potomac River and San Francisco Bay. The dispersion coefficients reported for these systems have been widely variable. In 1958, Kent (17) reported values of E ranging from 4 to 14 mi^2/day in the Delaware estuary. Work by O'Connor in 1963, 1965, and 1968 (18,19,20) on the Delaware suggests E to be in the 1 to 6 mi^2/day range. Harleman (14) however reports that the proper E range for the Delaware estuary is about 2 mi^2/day, while Thomann (21) is convinced that 2 to 5 mi^2/day is more appropriate. Recently, Paulson (22) proposed values of E in the range of 4 to 6 mi^2/day..

An examination of dispersion coefficients obtained for the Potomac estuary reveals an even wider variation. Custer and Krutchkoff (23) report E to be in the 2 to 5 mi^2/day range while Harleman (14) reports E to be in the 0.1 mi^2/day range. Prych and Childey (24) however have argued that E must be on the order of 1 mi^2/day for the Potomac.

Glenne (25) reports an extremely wide range of E values for the San Francisco estuary, .6 to 40 mi^2/day. Selleck (26) however recommends values of 1 to 3 mi^2/day for the San Francisco Bay. These latter values are accepted by the California Water Resources Board (27).

Dobbins (28), DiToro (29), Thomann (30) and O'Connor (31) agree that dispersion may be neglected for stream pollution models while Dresnack (32), Fischer (33), Harlemen (14), and, Thackston, Hays and Krenkel (16) feel that it should be included in these models. All researchers agree that longitudinal dispersion is of major significance in estuary modeling.

Several field tests were run in the course of this study to determine a value for the dispersion coefficient for a particular stream. The results indicate that the dispersion coefficients reported by many contemporary investigators are considerably larger than the values found for the test stream. The writer's results compare more favorably with Holley (8) who suggests dispersion coefficients for streams as low as 0.01 square miles per day.

CHAPTER 3 THEORY OF IMAGES

Image theory, also known as the reflection principle, is a well-known tool of engineering. The principle of reflection shall be used in the formulating of a method for determining dispersion and diffusion coefficients.

3.1. Theory

In many engineering problems for which a differential equation may be written, the imposition of essential boundary conditions may make it difficult or impossible to obtain an analytic solution of the equation. If an analytic solution to the unbounded equation is known it may be possible to recognize symmetries in the intractable problem and to formulate unbounded problems which have as a solution the solution to the intractable problem.

3.2. Application

A well established application of image theory is in the field of groundwater engineering. Here solutions to well flow problems are often limited by finite physical boundaries. For example, the differential equation governing steady radial flow to a discharging well in an unconfined isotropic homogeneous aquifer may be written (34) as

$$Q = -2\pi\, r\, K\, h\, \frac{dh}{dr} \qquad (17)$$

where Q = well flow rate

K = permeability of the aquifer

h = head above bedrock

$r = \sqrt{x^2 + y^2}$, radial distance from well

This first order differential equation is readily solved by separation of variables for the boundary conditions $h = h_w$ (head at the well) at $r = r_w$ (well radius) and $h = h_o$ (initial head) at $r = r_o$ (radial extent of the final depression cone). Integration of Equation 17 yields,

$$h_o^2 - h_w^2 \approx \frac{Q}{\pi K} \ln (r_o/r_w) \qquad (18)$$

Now consider a case where the depression cone is interrupted by a perennial stream as shown in Figure 8. Obviously the above solution is no longer valid. Furthermore the imposition of this new boundary condition creates difficulty in obtaining an analytical solution. However a simplified solution may be obtained by applying the method of images. In this case the image will be an imaginary recharge well introduced in such a way as to create, mathematically, a hydraulic flow system which will be equivalent to the known physical boundary of the actual flow system. Thus the aquifer of finite extent is transformed into one of infinite extent so that the known radial flow solution can be applied. The image well, Figure 8(b), is placed directly opposite and at the same distance from the stream as the real well. The image well operates simultaneously and at the same recharge rate as the real well. The net result of buildup from the image recharge well and drawdown from the real well is then combined graphically.

The final head may be predicted graphically by imposing the individual head profiles for each well as calculated by Equation 18. The resultant head at each point is the graphical sum. An even easier method is to write simply the equation which is the algebraic sum of the two governing equations.

$$(h^2 - h_w^2)_{res} = (h^2 - h_w^2)_{real} + (h^2 - h_w^2)_{image} \qquad (19)$$

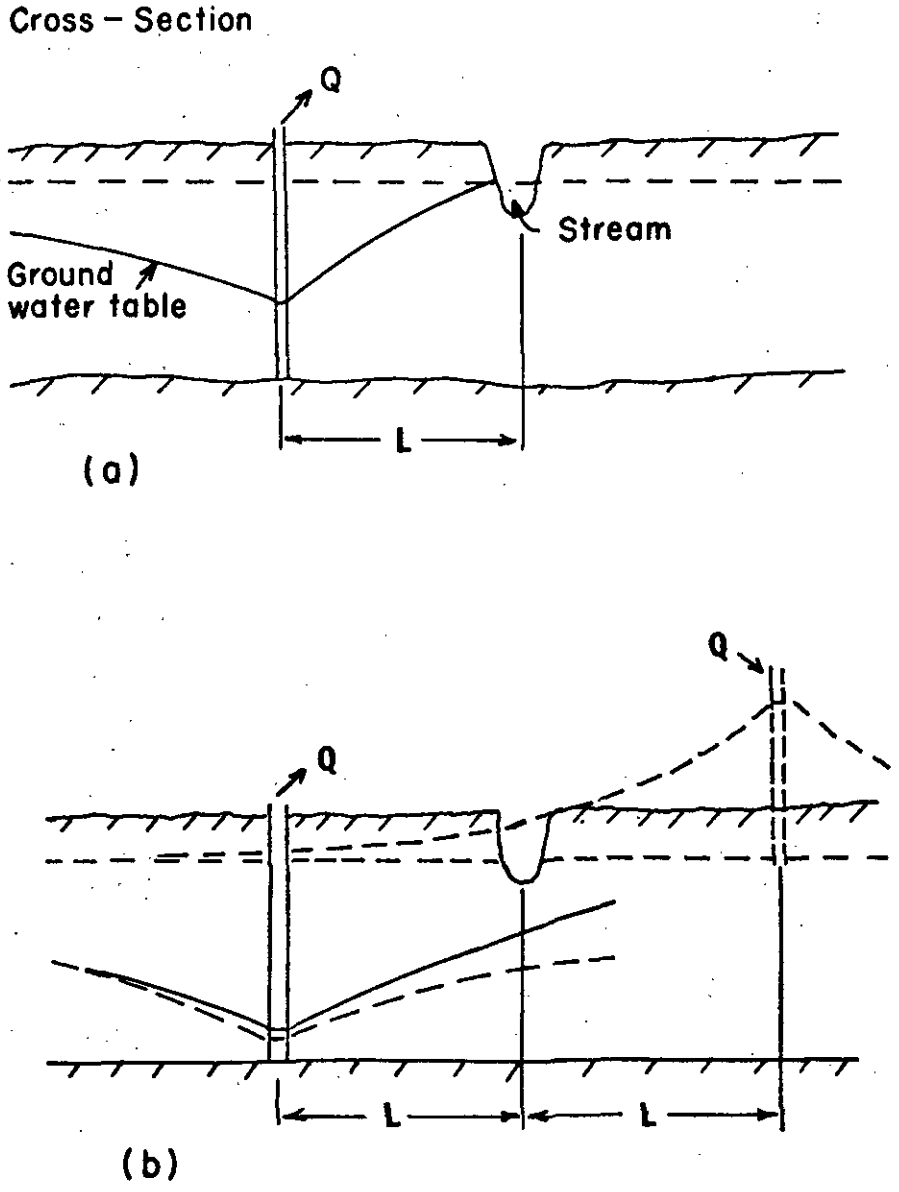


Figure 8. Sectional Views of A Discharging Well Near A Perennial Stream and Equivalent Hydraulic System in An Aquifer of Infinite Aerial Extent

where $h - h_w$ is the drawdown at any point (x,y). The coordinate x-axis is the line perpendicular to the stream going through the real well and the y-axis runs through the center of the stream.

Another common boundary problem in well flow calculations is that of an impermeable vertical boundary near a pumping well. This problem and its hydraulic equivalent are shown in Figure 9. For this case the boundary is replaced by an imaginary discharge well placed equidistant from the boundary and in the opposite direction from the real well. Thus the wells offset one another yielding no flow across the boundary. This is of course the desired condition.

As a further example of image theory, consider the emission of gases or aerosols from a stack as shown in Figure 10. This particular example is very closely related to the receiving stream and waste outfall system solution to be presented by this writer. The differential equation governing the mass transfer of air-borne materials emitted from a point source in an advective velocity field is similar to that given for the aqueous system, with the exception that longitudinal dispersion is neglected in view of the magnitude of the advective term. An analytical solution could be obtained for the steady-state problem were it not for the boundary condition imposed by the ground surface. Obviously the solution requires that no pollutant be allowed to pass through this boundary. The problem, however, can be solved by replacing the boundary with an image stack discharging at the same rate as the real stack. The imaginary stack is located equidistant but in the opposite direction from the boundary. A solution then is obtained simply by algebraically summing the resultant

CROSS - SECTION

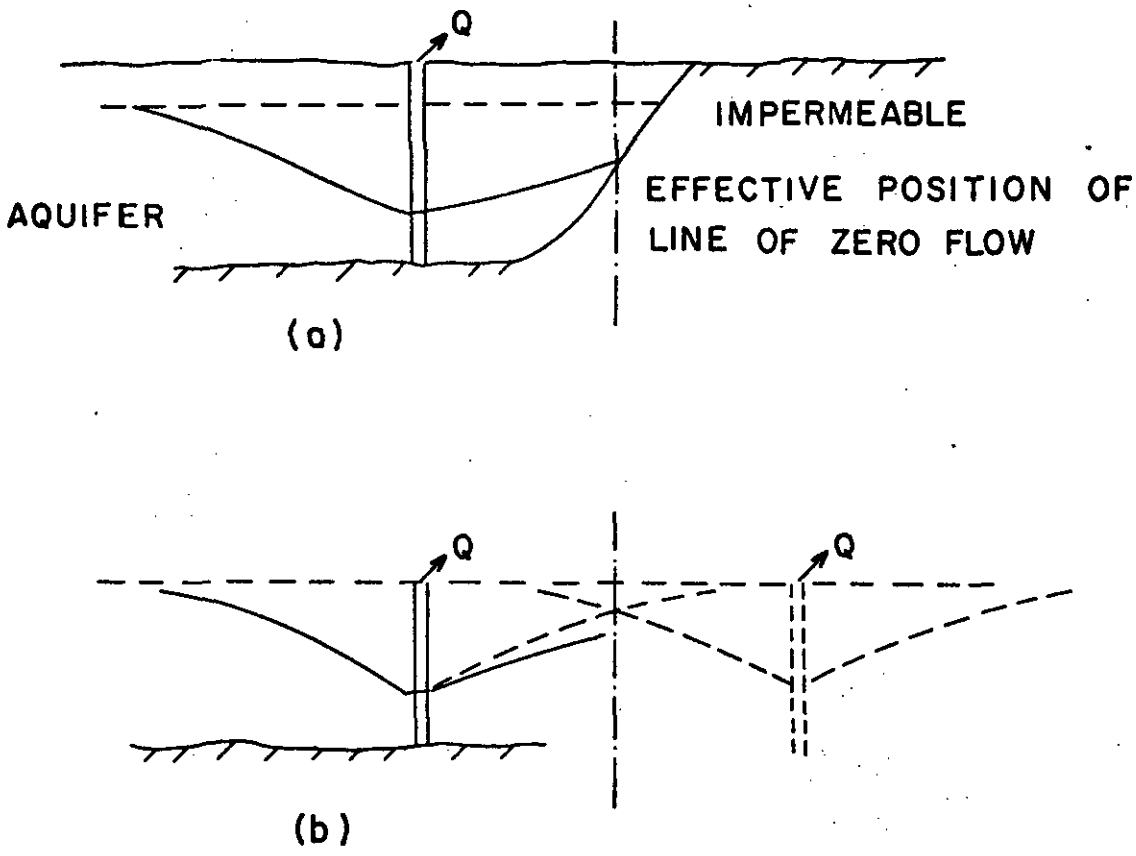


Figure 9. Sectional View of a Discharging Well Near An Impermeable Boundary and Equivalent Hydraulic System in An Aquifer of Infinite Aerial Extent.

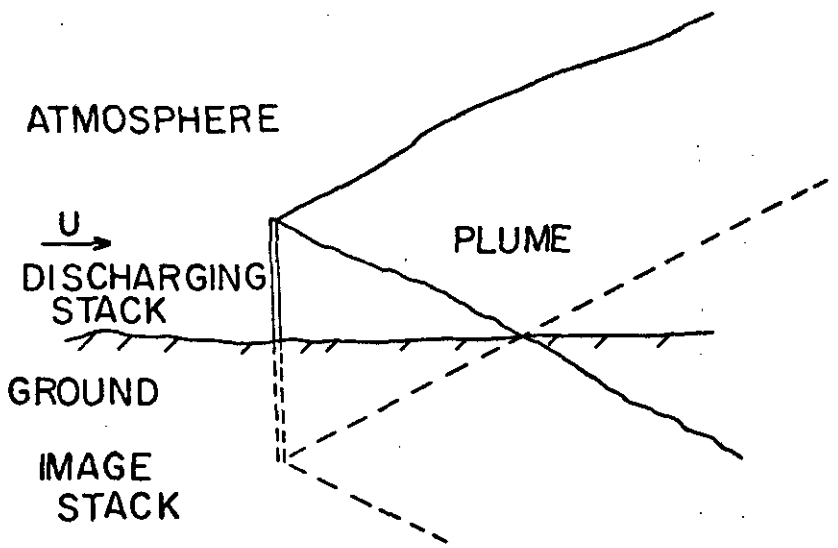


Figure 10. Sectional View of A Discharging Stack and An Image Stack Which Together Replace the Ground Surface Boundary.

concentrations due to each of these sources. The solution is given by Turner (10) as

$$\bar{c}\ (x,y,3) = \frac{Q}{2\pi\ \sigma_y \sigma_z\ U}\ e^{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2}\ e^{-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2} + e^{-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2}$$

where σ_y and σ_z are the standard deviations of the plume in the lateral and vertical directions. The standard deviations are related directly to the turbulent diffusion coefficient, normally designated K in atmospheric dispersion investigations, by $\sigma = \sqrt{2Kt}$. The concentration of suspended material is assumed to have a Gaussian distribution in both horizontal and vertial directions. Turbulent dispersion in the longitudinal direction is neglected in view of the overwhelming effect of the mean wind speed, U. This assumption is quite logical in view of the previous discussion (Section 2.6). Adrian (35) has proposed a similar solution for the atmospheric dispersion problem involving both an upper (thermal inversion) boundary and lower (ground surface) boundary.

The reflection principle is also used in the solution of certain hydrodynamic problems. Again boundaries are accounted for by selectively placed images which allow one to work with known unbounded analytical solutions. An elementary example of this use is that of a jet flowing against a flat plate. In two dimensions the jet may be represented by a singular source and the plate by a straight line. In polar coordinates the radial velocity for unbounded source flow of strength $\lambda = 2\pi r q_r$, is

$$qr = \frac{\partial \psi}{r \partial \theta}$$

where ψ is the stream function. If the polar coordinate origin is picked to be coincidental with the source and $\psi = 0$ to be the horizontal stream function then the solution is readily found (36) to be

$$\psi = \frac{\lambda \theta}{r\pi}$$

Unfortunately the solution for the actual bounded case is considerably more difficult with the imposition of $\psi = 0$ and $\frac{\partial \phi}{\partial \eta} = 0$ for the plate. However by applying an imaginary external sigularity equidistant from the plate a solution is easily obtained. This solution is simply the superposition of the real and imaginary sources in an unbounded infinite field (36). It should be noted that continuity requires there must likewise be imposed real and imaginary sources at $+\infty$ and $-\infty$, although this is not significant in the solution to this problem.

There are many other areas where image theory is applied. Various electrical, acoustic and heat transfer problems are often solved using reflection principles. In many cases the problems to be solved are considerably more complex and their image theory solutions may involve numerous imaginary systems.

CHAPTER 4 COMBINED FORMULATION

Having considered the theoretical mechanics of mixing and advection in Chapter 2 and the method of images in chapter 3, a mathematical solution to the two and three-dimensional mixing-advective equation is now presented. As already discussed, the one-dimensional problem is readily solved analytically due to the infinite nature of the longitudinal boundary. That is, the stream may be considered to be unbounded in the upstream and downstream direction. However, in the two- and three-dimensional mixing cases, finite boundaries do exist as represented by the surface, bottom, and banks of a natural stream. This situation, with finite boundary conditions in the vertical and lateral directions, has left the three-dimensional flow equation without an analytic solution. In this section is proposed a solution for these bounded flow cases by applying the principles of image theory. The problem will first be considered graphically and then mathematically.

4.1. Graphical Description

The method of images in solving the boundary value problem of three-dimensional mixing-advective flow is presented graphically in Figure 4.1. The plume boundaries emanating from the real point source outfall may be defined arbitrarily. For instance the plume surface might be defined as the connection of points whose concentration is one standard deviation away from the maximum of the normal concentration profile. In Figure 11(a) the first image source is placed in such a manner as to replace the real effect of the right bank boundary. The boundary effect is of course one

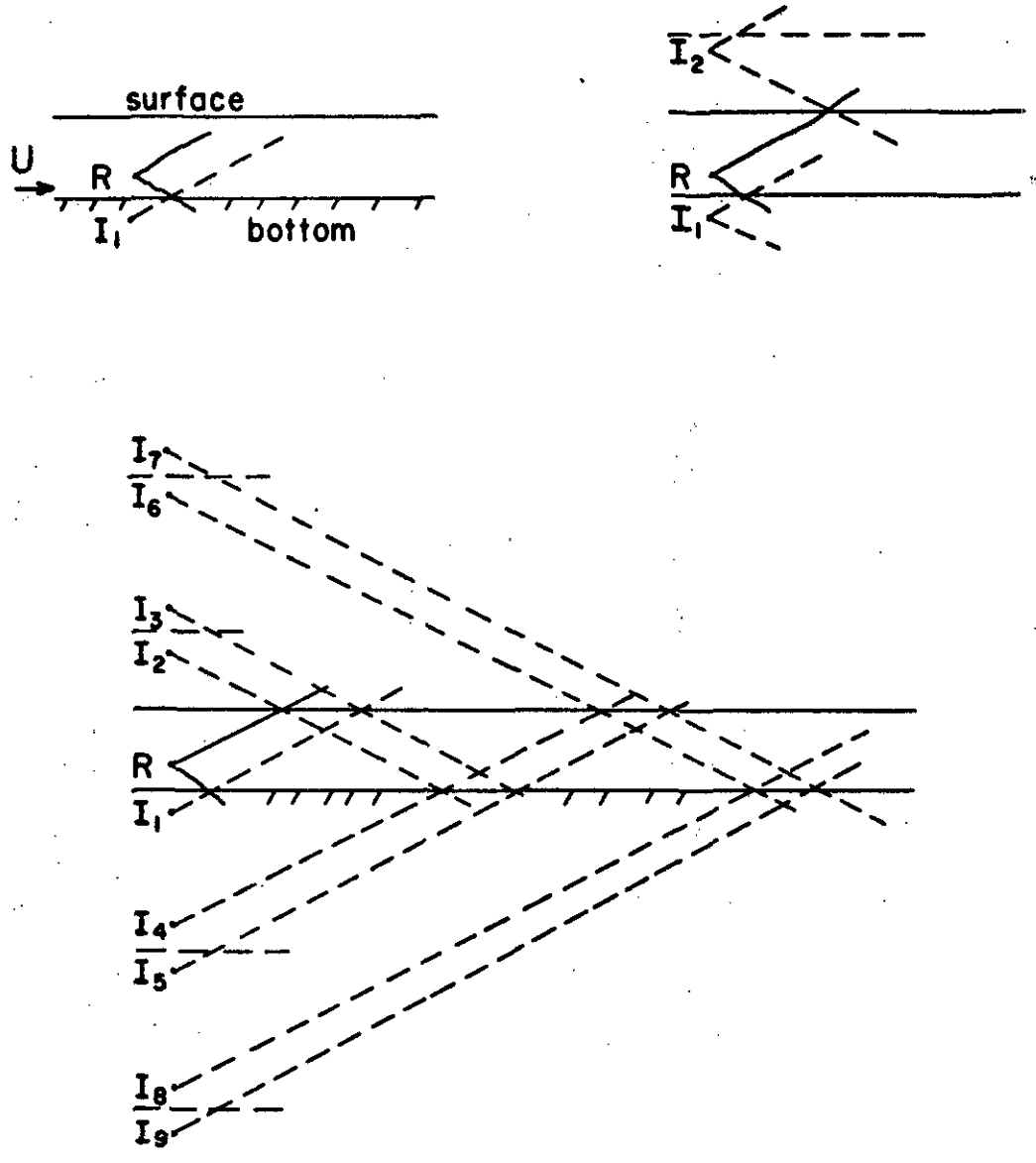


Figure 11. Aerial View of Stream Illustrating the Progressive Superposition of Imaginary Plumes to Accomplish the Image Method of Solution.

of reflection (assuming no adsorption onto the boundary). The net effect of replacing the boundary with an imaginary source is precisely the same. The total mass to be found inside of the real boundaries will be consistent with the amount actually being released. Now the effect of the real plume must also be accounted for at the left bank boundary. This situation is accounted for in Figure 11(b) by placing an image source, l_2, equidistant from the real source about the left bank boundary. However, the solution is still incomplete as the l_1 plume soon intersects the left bank, necessitating the introduction of l_3 as shown in Figure 11(c). Similarly, the l_2 plume intersecting the right bank must be reflected back into the stream by introducing the imaginary source l_4. In a like manner, when the plumes of l_3 and l_4 intersect the distant boundaries, two more images l_5 and l_6 will be required. Theoretically, an infinite number of images will ultimately be required in order to satisfy the condition of mass conservation. In practice, however, one will note that the effect of images spaced at increasing distances from the stream will be of less and less importance. This is especially true immediately below the outfall and is shown graphically by noting that points just downstream of the outfall are within only the real and l_1 plume. Points further downstream are affected by the real plus only four or five image plumes. As one proceeds downstream more images will be required. It is still evident however that an infinite number of images will not be required.

The discussion thus far has considered only a method for coping with the horizontal boundaries of a stream. The two vertical stream boundaries,

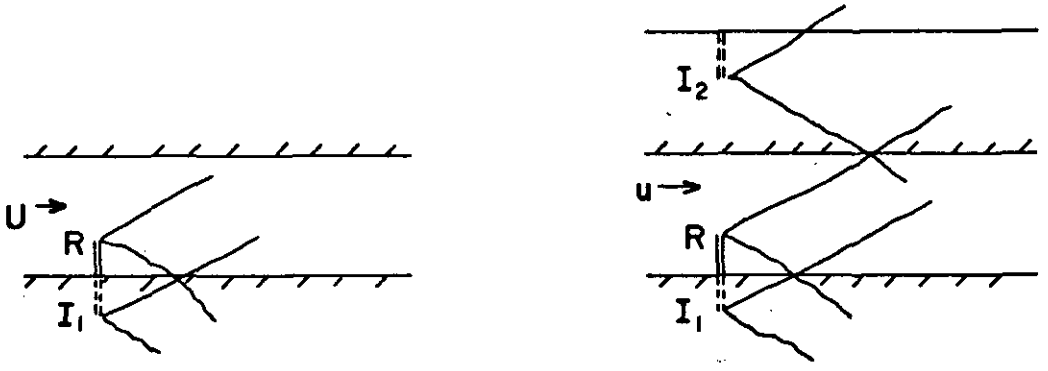
the air-water interface and the bottom of stream, present a similar problem and it is proposed here that these boundaries may be treated in a similar manner. Figure 12 illustrates how the surface and bottom boundaries are accounted for graphically. Again, mass is conserved by distributing imaginary sources spaced appropriately about the surface and bottom stream boundaries. One will note here that since the magnitude of stream depth is considerably less than stream width several more images are required in order to satisfy mass conservation even at points very near the real outfall. The result of this relatively tight spacing is a rapidly developed uniform concentration profile in the vertical direction. This is precisely the result expected according to the earlier discussion of turbulent mixing, Section 2.1. Thus this method of treating the boundary conditions is consistent with multidimensional mixing concepts.

4.2. Mathematical Description

The differential equation governing three-dimensional mixing in a uniform flow field may be written as

$$\frac{\partial \bar{c}}{\partial t} + U \frac{\partial \bar{c}}{\partial x} = E \frac{\partial^2 \bar{c}}{\partial x^2} + D_y \frac{\partial^2 \bar{c}}{\partial y^2} + D_z \frac{\partial^2 \bar{c}}{\partial z^2} + M\delta(x-x')\ \delta(y-y')\ \delta(z-z')\ \delta(t-\tau) \quad (21)$$

The symbol E denotes the coefficient of longitudinal dispersion which was defined earlier as the net effect of the random longitudinal diffusion and the non-random longitudinal dispersion process. The symbols D_y and D_z are coefficients of lateral and vertical turbulent diffusion and are considered to be random processes. The Dirac delta function, δ, relates the instantaneous input of mass, M, with the time of mass release, τ, relative to a fixed time, and the coordinate locations at the point of mass release x', y', z'. The following boundary conditions must be satisfied in



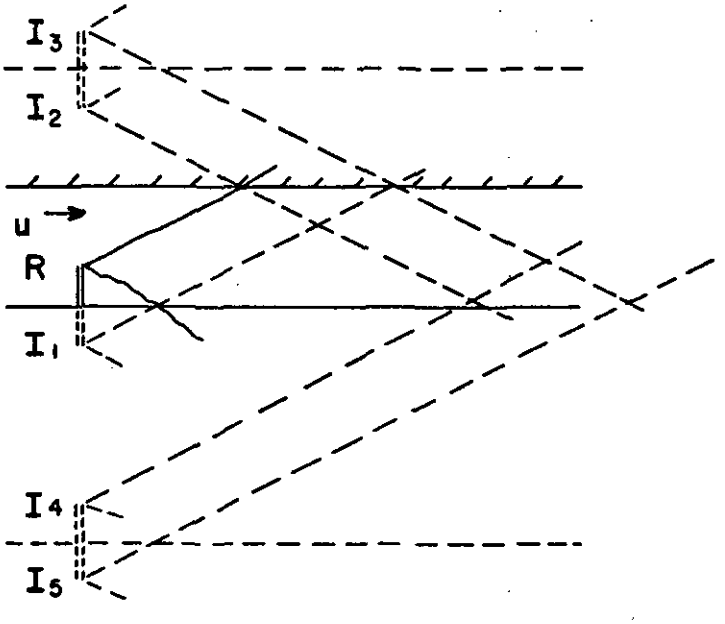


Figure 12. Cross-sectional View of A Stream Illustrating the Progressive Superposition of Imaginary Plumes to Accomplish the Image Method of Solution.

obtaining an analytical solution to Equation 21 describing three-dimensional mixing and advective transport of the system.

I.C. $\bar{c}\ (x,y,z,o) = 0$ for $-\infty < x < +\infty$, $o \leq y \leq w$ and $o \leq z \leq D$

B.C. $\frac{\partial \bar{c}}{\partial y} = 0$ for $y = 0$, $t > 0$ and for $y = w$, $t > 0$

$\frac{\partial \bar{c}}{\partial z} = 0$ for $z = 0$, $t > 0$ and for $z = D$, $t > 0$

$\bar{c} \to 0$ as $x \to \pm \infty$

An analytic solution has not as yet been obtained which will satisfy Equation 21 with the above boundary conditions.

The unsteady-state solution to Equation 21 for the case of infinite lateral, vertical, and longitudinal boundaries is adapted from the solution given by Carslaw (15) as

$$\bar{c} = \frac{M}{\{2\sqrt{\pi(ED_yD_z)^{1/3}t}\}^3} \exp -\frac{1}{4t}\left\{\frac{(x-x'-Ut)^2}{E} + \frac{(y-y')^2}{D_z} + \frac{(z-z')^2}{D_z}\right\}$$

For this equation $\bar{c}$ is the time average concentration resulting at any x, y, z, point from the release of a mass, M, at time $t = 0$ at the rectangular coordinate point (x', y', z'). If the exact location of the mass injection is picked as the origin then the equation may be simplified to

$$\bar{c} = \frac{M}{\{2\sqrt{\pi(ED_yD_z)^{1/3}t}\}^3} \exp -\frac{1}{4t}\left\{\frac{(x-Ut)^2}{E} + \frac{y^2}{D_y} + \frac{Z^2}{D_z}\right\} \qquad (23)$$

The above equation then predicts the concentration at any location and time resulting from an instantaneous injection of mass into an unbounded advective turbulent field. Now to obtain a solution for the actual finite boundary value problem, a series of imaginary outfalls will be mathematically superimposed upon the real outfall. The superposition is analogous to the graphical description given in Section 4.1. Mathematically, the solution is obtained by evaluating Equation 21 for a finite number of imaginary sources and summing up the resultant concentrations due to each individual outfall. The number of imaginary sources needed will be dictated by the accuracy desired since each additional image source will contribute less mass than the previous one.

The computations to be made will take the form shown below. Figure 13 illustrates the symbols used for the hydraulic stream parameters with the actual outfall located at the origin.

Concentration due to real outfall

$$\bar{c}_R = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{E D_y D_z}} \exp - \frac{1}{4t} \left| \frac{(x-Ut)^2}{E} + \frac{y^2}{D_y} + \frac{z^2}{D_3} \right|$$

Concentrations due to vertical image outfalls (z axis)

$$\bar{c}_{v_1} = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{E D_y D_z}} \exp - \frac{1}{4t} \left\{ \frac{(x-Ut)^2}{E} + \frac{y^2}{D_y} + \frac{(Z-2d)^2}{D_z} \right\}$$

$$\bar{c}_{v_2} = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{E D_y D_z}} \exp - \frac{1}{4t} \left\{ \frac{(x-Ut)^2}{E} + \frac{y^2}{D_y} + \frac{(Z-2d+2D)^2}{D_z} \right\}$$

$$\bar{c}_{v_3} = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{E D_y D_z}} \exp - \frac{1}{4t} \left\{ \frac{x-Ut)^2}{E} + \frac{y^2}{D_y} + \frac{(Z+2D)^2}{D_z} \right\}$$

etc.

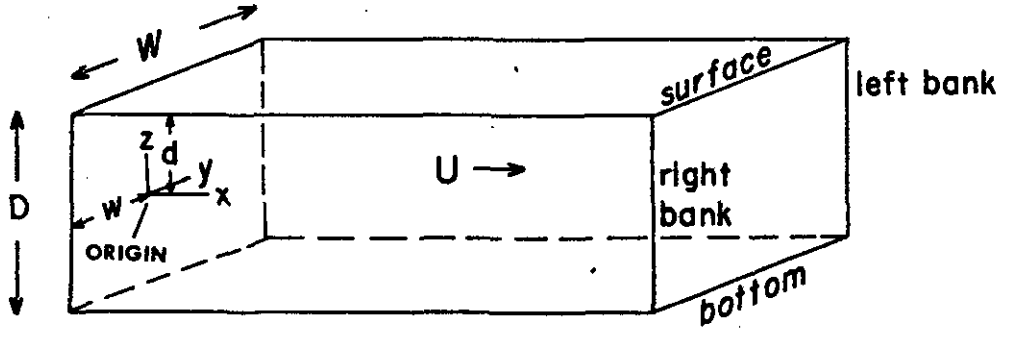


Figure 13. Drawing of A Rectangular Stream Channel Showing Symbols Used to Identify Physical Parameters.

Concentrations due to horizontal (y axis) image outfalls

$$C_{H_1} = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{ED_yD_z}} \exp - \frac{1}{4t}\left\{\frac{(x-Ut)^2}{E} + \frac{(y-Zw)^2}{D_y} + \frac{Z^2}{D_z}\right\}$$

$$C_{H_2} = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{ED_yD_3}} \exp - \frac{1}{4t}\left\{\frac{(x-Ut)^2}{E} + \frac{(y-2W+2w)^2}{D_y} + \frac{Z^2}{D_z}\right\}$$

$$C_{H_3} = \frac{M}{(2\sqrt{\pi t})^3 \sqrt{ED_yD_z}} \exp - \frac{1}{4t}\left\{\frac{(x-Ut)^2}{E} + \frac{(y+2W)^2}{D_y} + \frac{Z^2}{D_z}\right\}$$

etc.

Total concentration at any point

$$\bar{c}_T = \bar{c}_R + \sum_{i=1}^{n} \bar{c}_{H_i} + \sum_{i=1}^{m} \bar{c}_{v_i}$$

Thus the resultant concentration at any point in the stream following an instantaneous release of mass may be calculated. The symbols used (with their dimensions) are defined as follows:

M = mass of material released. (M)

E = coefficient of longitudinal dispersion. (L^2/T)

D_y, D_z = coefficients of lateral and vertical diffusion. (L^2/T)

t = time following release. (T)

x, y, z = cartesian coordinate location of point where concentration is to be predicted. (L, L, L)

U = average cross-sectional stream velocity for reach. (L/T)

d = depth of outfall below stream surface. (L)

w = width from left bank of outfall point. (L)

D = average stream depth. (L)

W = average stream width. (L)

C_R = mass concentration in stream at (x,y,z) due to real outfall. (M/L^3)

C_{V_1} = mass concentration in stream at (x,y,z,t) due to first vertical (z axis) image outfall. (M/L^2)

C_{H_1} = mass concentration in stream at (x,y,z,t) due to first horizontal (y axis) image. (M/L^3)

C_T = mass concentration at (x,y,z,t) due to all image sources. (M/L^3)

4.3. Significance of Vertical Mixing

Having presented a method for solving the three-dimensional problem, it is now appropriate to evaluate the need for this tedious approach. The three dimensional solution is useful only in the three-dimensional mixing zone. If this zone is resolved to be relatively small for typical stream and estuary conditions, then the need for maintaining the zone in the analysis may be justifiably dismissed. The necessity of continuing to include this zone will be demonstrated only if the magnitude of the three dimensional mixing length is significant.

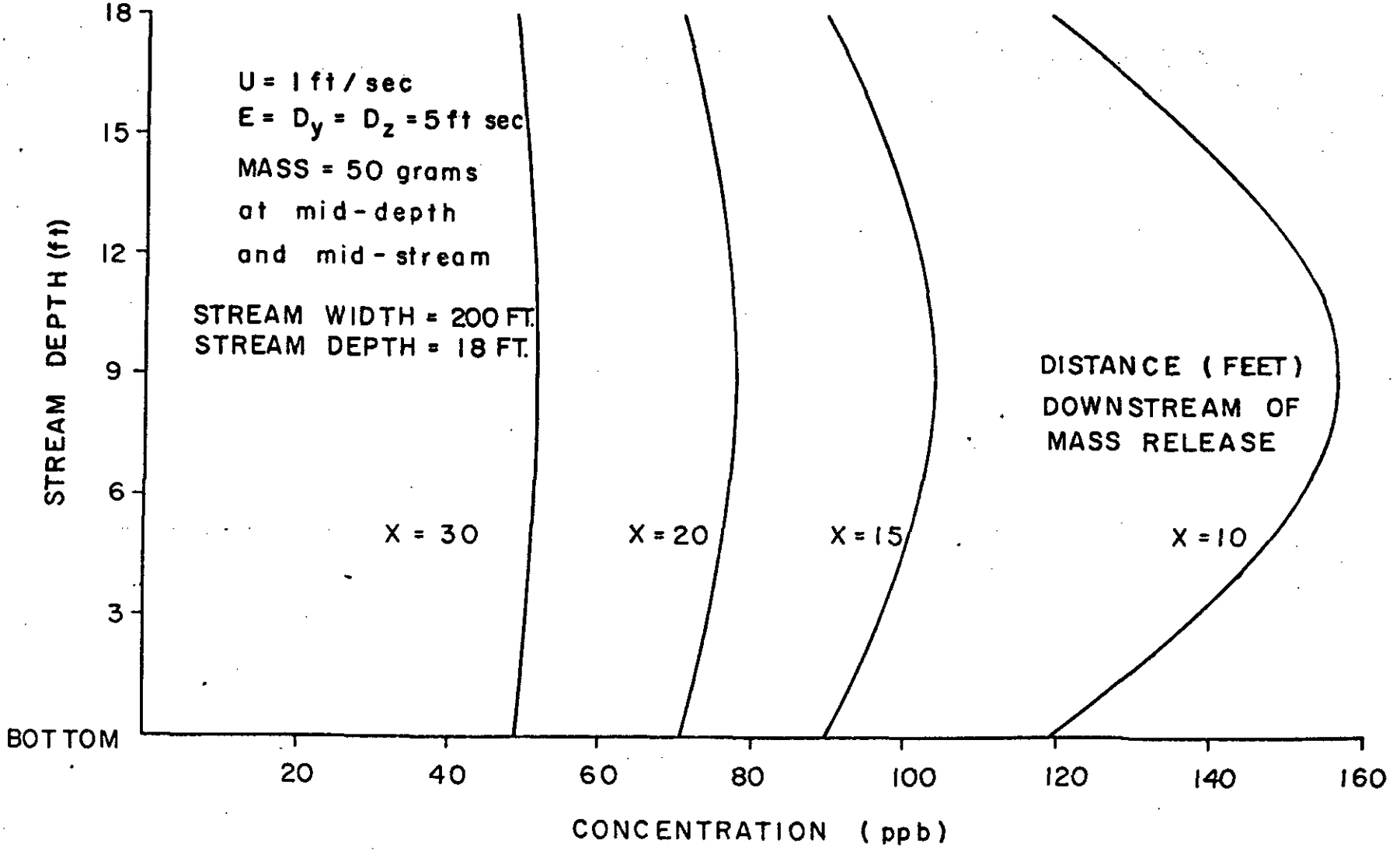
Using the image method in conjunction with various typical stream parameters, the extent of the three-dimensional mixing length can be examined. It should be recalled here that the lower boundary of this zone is designated by the appearance of vertical concentration uniformity

across this lateral boundary. For a given set of stream parameters, vertical concentration profiles may be calculated at any position downstream following an instantaneous release of mass. By adopting a Lagrangian viewpoint, that is by following the center of mass downstream, the extent of vertical mixing is seen under peak concentration conditions. The criteria for uniformity of mixing may be established at any level desired.

The results of a typical case are given in Figure 14. These curves depict the theoretical concentration profiles which would occur from a Lagrangian viewpoint due to a mass release of 50 grams at midstream and mid-depth. The hypothetical stream had a width of 200 feet, depth of 18 feet, mean velocity of 1 ft/sec., and it was assumed that $D_x = D_y = E = 5$ ft^2/sec. This figure shows the very rapid convergence of the vertical profiles as the mass moves downstream. All points on the profile converge to within 6% in a distance of 30 feet downstream of the mass injection.

The vertical mixing length is directly related (theoretically) to stream depth. This parameter is also a function of stream velocity, width, channel roughness, etc. Since the depth of a natural stream is consistently an order of magnitude less than the width, it follows that the vertical mixing length will be of far less significance than the lateral mixing length. This then suggests that the three-dimensional mixing length can be neglected in all but the most sensitive studies where vertical mixing is a critical factor. Such a situation might occur under an estuarine condition where stratification was non-limiting or

Figure 14. Vertical Concentration Profiles Predicted for Instantaneous Mass Release Indicating Rapid Convergence Downstream.



insignificant. However, estuaries are usually stratified due to saline or thermal differences between stream and ocean.

With the above discussion in mind, the complex three-dimensional mixing-advection model presented may be simplified to a more negotiable, *two-dimensional form by assuming that the effect of vertical mixing may* be neglected in the more general three dimensional case.

4.4. Two-dimensional Mixing-Advection Method

By neglecting vertical diffusion the differential equation governing the mass transport of material from an outfall becomes

$$\frac{\partial \bar{c}}{\partial t} + U\frac{\partial \bar{c}}{\partial x} = E\,\frac{\partial^2 \bar{c}}{\partial x^2} + D_y\,\frac{\partial^2 \bar{c}}{\partial y^2} \tag{24}$$

The solution for an instantaneous release of mass into the unbounded system is (15)

$$\bar{c}_R = \frac{M}{4\pi dt\sqrt{ED_y}}\exp - \frac{1}{4t}\left\{\frac{(x-Ut)^2}{E} + \frac{y^2}{D_y}\right\} \tag{25}$$

where d is the average stream depth. Again, the image method is used to satisfy the finite extent of the lateral boundaries. The series solution will then take the form

$$\bar{c}_T\,(x,y,t) = \bar{c}_R + \sum_{i=1}^{n} \bar{c}_{H_i} \tag{26}$$

The number of horizontal image outfalls will be determined by the accuracy desired. The horizontal image spacing is the same as that shown in Figure 11.

4.4.1. Application

The primary purpose for formulating the two-dimensional mixing-advection model is to use the model for evaluating specific values of the longitudinal dispersion and lateral diffusion coefficients. The means by which this can be accomplished is to create a situation where all parameters of Equation 22 are known except E and D_y. Prospects for solution of a problem of one equation and two unknowns initially look bleak; however, one property of this equation will allow another relationship to be established between D_y and E. That is, for a given value of E only one possibility for D_y exists. With this information E and D_y may be determined by trial and error substitution into Equation 26.

The longitudinal dispersion coefficient and the lateral diffusion coefficient can be evaluated by the image method for a specific stream reach by means of a tracer injection. A known quantity of mass is released and time-concentration profiles are measured at various downstream locations. Readily obtainable parameters including the average stream depth, width, and velocity must also be observed for the test. Ideally, a reach would be selected which is relatively straight with steep banks.

For an instantaneous mass release at mid-stream, the maximum concnetration found on any cross-section downstream would occur at y = 0 and x = Ut. Hence the total image series solution takes the form

$$\bar{c}_{max}(x,o,t) = \frac{M}{4td\pi\sqrt{ED_y}} \{1 + 2 \sum_{j=1}^{n} \exp - \frac{(jW)^2}{4D_y t} \} \qquad (27)$$

Now for a given value of E, a value of D_y may be sought by trial and error substitution into Equation 27. A family of E with corresponding values of D_y may thus be generated. With this information the time-concentration profile observed in a test may be fitted by trying various values of E and the corresponding values of D_y in Equation 27. Best fit values of E and D_y are thus obtained when the theoretical time-concentration profile reproduces the observed time-concentration profile within a given tolerance.

The mathematical calculations required in applying this method are far too extensive to be treated manually. The calculations are, however, extremely well-suited to electronic digital computation. In fact, one might speculate that the image method of solution has likely been recognized by earlier investigators but dismissed due to the extensive computations needed. There is no question that the image method is of little value without the digital computer.

4.4.2. Data Analysis

The method of solution of Equation 26 for the bounded two-dimensional mixing-advection problem does not allow direct calculation of E. Furthermore, experimental error is introduced in several ways when the theoretical method is applied in the field. In searching for a value for E by trial and error one must expect that the E obtained will be the result of a best fit analysis. The most common procedure for obtaining a best fit is to minimize the sum of the squared differences between C_p (predicted or theoretical concentration) and C_o (observed concentration). Two methods for accomplishing the best fit analysis

are discussed in the sections to follow.

GRADIENT METHOD

The gradient method is a form of least squares analysis in which values of two independent variables occurring in a single equation may be obtained by minimizing the difference between a set of predicted and observed values of a dependent variable. The observed value set as obtained experimentally, or in this case from field data, while the predicted value set is calculated for various assigned values of the unknown variables. The procedure requires repetitive calculations and is well adapted for computer analysis. The method proceeds as follows:

1. Starting from initial values of E and D_y new values are calculated by increasing and decreasing each value by an incremental amount resulting in a three by three array of E and D_y values. Incremental values need not be the same magnitude for each variable.
2. The sums of squared differences (SSD) between the observed and the predicted concentration values are calculated for each of the nine pairs of E and D_y values.
3. SSD values for each network point out are compared with the central point of the array and the smallest SSD is determined. This point is then established as the central point of a new array of E and D_y values.

4. Steps 2 and 3 are then repeated until the central point is determined to be the data pair yielding the minimum SSD and the procedure is terminated. This final data pair is thus selected as the best fit value of E and D_y for the experimental data. Smaller increments may be assigned to E and D_y and the procedure may be repeated starting with Step 1 to obtain more accurate values.

NEWTON-RAPHSON ANALYSIS

An alternate method for selecting best fit values for E and D_y is the more sophisticated Newton-Raphson analysis. This method is also a least squares minimization technique, however this method allows a much more direct and thus more rapid analysis. For the one-dimensional mixing problem where only a value of E is sought, the procedure is as follows:

$$\text{Let } G(E) = \Sigma\ (\bar{c}_o - \bar{c}_p)^2$$

$$\text{and } H(E) = \frac{\partial G}{\partial E} = 2\ \Sigma\ \left\{ (c_o - c_p)\ \frac{\partial \bar{c}_p}{\partial E} \right\}$$

Now the E value sought is that value for which

$$\frac{\partial G}{\partial E} = 0$$

This particular value for E is designated E_o. Figure 15 graphically illustrates the above mathematical reasoning. In order to find the optimum value, E_o, the function H(E) is expanded by the Taylor Series around E_o and truncated after the first term.

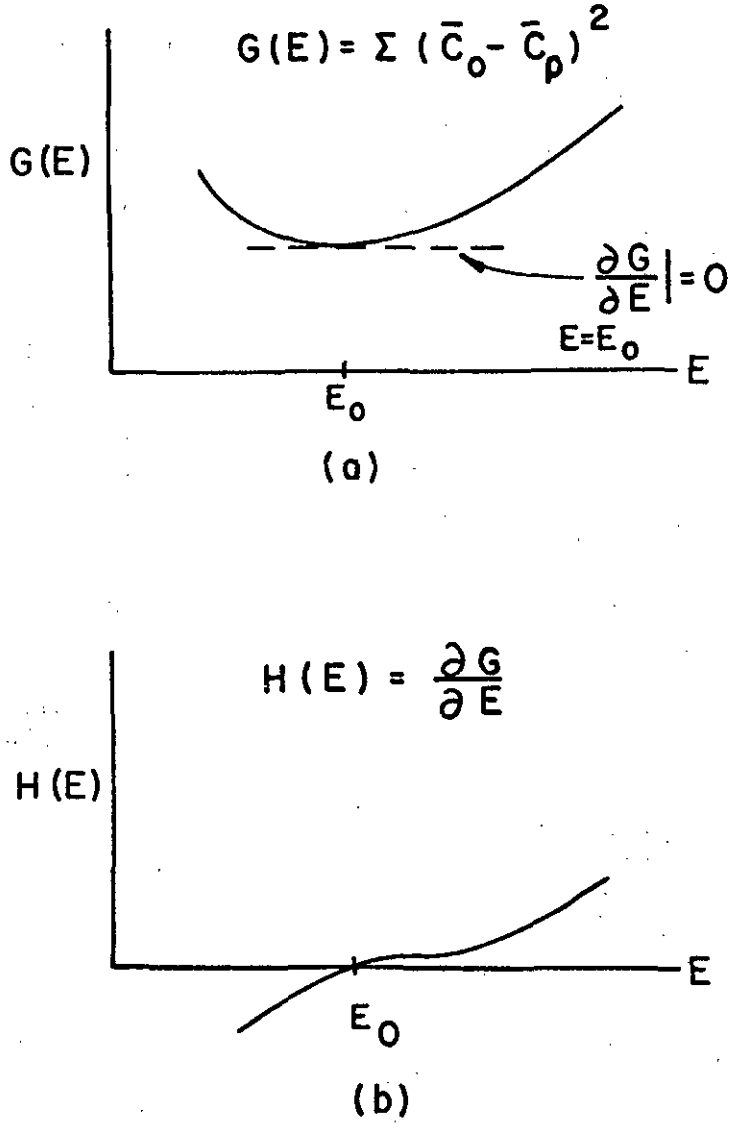


Figure 15. Graphical Description of the Newton-Raphson Method for Determining Optimum Dispersion Coefficient E_o from Experimental Data in One Dimensional Mixing Zone.

$$H(E) = H(E_o) + \frac{\partial H}{\partial E}\Big|_{E=E_o} (E-E_o)$$

Now since $H(E_o) = 0$, an arbitrary initial value of E may be assigned and the above equation may then be solved algebraically giving a better approximation for E. Now the procedure is repeated utilizing the refined approximation of E. Successive improved approximations continue to be made until sufficient convergence is obtained. The final value of E, is accepted as the optimum value, E_o of the dispersion coefficient.

For the two-dimensional mixing-advection equation the Newton-Raphson procedure requires several differentiations with respect to E and D_y. In general, the two-dimensional series solution may be written as follows:

$$\bar{c}_p(x,y,t) = \frac{M}{4\pi dt\sqrt{ED_y}} \{\exp - \frac{1}{4t}\left[\frac{(x-Ut)^2}{E} + \frac{y^2}{D_y}\right] +$$

$$\exp - \frac{1}{4t}\left[\frac{(x-Ut)^2}{E} + \frac{(y-2w)^2}{\quad}\right]\}$$

$$\sum_{j=1}^{k} \sum_{n=1}^{4} \frac{M}{4\pi dt\sqrt{ED_y}} \exp - \frac{1}{4t} \{\frac{(x-Ut)^2}{E} +$$

$$\left[\frac{y + 2\sqrt{2}jW\sin(2n-1)\frac{}{4} - w\,(-1^n + 1)}{D_y}\right] \}$$

where the value of k relates the number of image sources desired. Again W is the stream width and w is lateral distance from either bank to the point of mass release. An obvious simplification of this expression may be obtained by specifying that w be fixed at W/2, i.e. the mass is to be released at mid-channel. This assignment reduces the solution to:

$$\bar{c}_p\ (x,y,t,) = \sum_{j=0}^{k} \sum_{n=1}^{4} \frac{M}{4\pi dt\ \sqrt{ED_y}} \exp - \frac{1}{4t} \left\{ \frac{(x-Ut)^2}{E} + \frac{[y+jW(-1)^n]^2}{D_y} \right\}$$

Now by substituting the following expressions

$$A_1 = \frac{M}{4\pi dt}$$

$$A_2 = -\frac{1}{4t}\ (x-Ut)^2$$

$$A_3 = -\frac{1}{4t}\ [y+jW(-1)^n]^2$$

the equation simplifies to:

$$\bar{c}_p = \sum_{j=0}^{h} \sum_{h=1}^{4} \frac{A_1}{\sqrt{EDy}} \exp\ \left(\frac{A_2}{E} + \frac{A_3}{D_y}\right)$$

Then G(E, Dy) will be defined by

$$G(E, D_y) = \sum_{l=1}^{s} [\bar{c}_o - \Sigma\Sigma \frac{A_1}{\sqrt{ED_y}} \exp\ \left(\frac{A_2}{E} + \frac{A_3}{D_y}\right)]^2$$

where s = number of data points

The functions f and h are then defined as

$$f = \frac{\partial G}{\partial E} \quad \textit{and} \quad h = \frac{\partial G}{\partial D_y}$$

Expanding these functions by the Taylor series yields the following equations

$$f(E, D_y) = f(\alpha,\beta) + \frac{\partial f}{\partial E}(E-\alpha) + \frac{\partial f}{\partial D_y}(D_y-\beta)$$

$$h(E, D_y) = h(\alpha,\beta) + \frac{\partial h}{\partial E}(E-\alpha) + \frac{\partial A}{\partial D_y}(D_y-\beta)$$

where α and β are optimum values of the dispersion and diffusion coefficients, E and D_y. Note that when α and β are correctly located the terms $f(\alpha,\beta)$ and $h(\alpha,\beta)$ both equal to zero. The pair of equations can thus be solved for α and β in the following manner. Initial values for E and D_y are assigned and the expressions for $\frac{\partial f}{\partial E}$ and $\frac{\partial f}{\partial D_y}$ are evaluated at the point (E, D_y). The expressions $\frac{\partial h}{\partial E}$ and $\frac{\partial h}{\partial D_y}$ are also evaluated at the point (E, D_y). Now the pair of equations are solved simultanteously for the values of α and β . These values obtained are first approximations. The function G may now be evaluated at the points (E, D_y) and (α,β) as a measure of the accuracy of the first approximation. Now letting

$$E = \alpha$$

and

$$D_y = \beta$$

the simultaneous equations may be resolved and a second approximation of α and β obtained. The process is repeated until sufficient convergence is obtained.

Although the method appears somewhat straight forward, the mathematics involved are extremely tedious. Mr. Thomas Sanders, Doctoral Candidate, Department of Civil Engineering performed the analysis described above and adapted the method for electronic digital computation. The program used for the analysis is given in the appendix.

CHAPTER 5 FIELD STUDY

Having proposed a theoretical solution to the two-dimensional mixing-advection equation, the writer conducted two field tests in order to determine the dispersion and diffusion coefficients for an actual case and to compare the results with published values.

5.1. Procedure

A reach of the Mill River in Northampton, a minor tributary of the Connecticut River was chosen for the field tests. The reach chosen was extremely straight longitudinally and trapezoidal in cross-section. The flat sandy river bottom was banked by rip-rap which had been placed to stabilize the channel. Admittedly, the test conditions were most ideal. Three tests were made during May and June, 1970. The discharge of the river during the tests ranged from 70 to 150 cfs. The stream width was 50 feet and the depth was 2 1/2 to 3 feet; thus the average velocities were about 0.7 to 1.0 fps. Figure 18 is a photograph of the test reach.

The tracer used in the tests was a well-known tracer used in stream studies (37), Rhodamine B. This organic dye was purchased from Fisher Scientific Company in powdered form. The dye was dissolved in a 40% (by weight) solution of glacial acetic acid just prior to each test run. Approximately 100 grams of dye were used in each test.

The analyses were performed with an Aminco-Bowman Spectrophotofluorometer (Silver Springs, Maryland--Catalog No. 4-8202). This highly sensitive instrument features a high intensity xenon light source with two monochromators. The excitation and emission spectra were scanned for the dye used and results verified those reported in

the literature (38). An excitation wave length of 546 mμ and an emission wave length of 570 mμ were used for all analyses. The instrument is capable of measuring concentrations down to 10 ppb (parts per billion) of Rhodamine B dye. Standards were prepared using the river water for dilution. Calibration curves were prepared by plotting log concentration versus log % transmittance for the appropriate photomultiplier settings.

The first test was run primarily as a reconnaissance survey in order to identify unforeseen complications. Since the method of solution presupposes two-dimensional mixing, the dye injection must be made uniformly in the vertical direction. To accomplish this, the dye solution was injected into the wake of a person standing in the center of the stream. The idea was to create excessive but short-lived turbulence to accomplish immediate vertical mixing. Unfortunately, the dye remained for a short period of time in the eddies created behind the stationary figure producing a visible tail as the dye mass proceeded downstream. In the second test, the problem was overcome by releasing the dye solution from a coffee can held below the surface. The cylindrical can was turned into the flow and the top and bottom plastic covers were quickly removed.

Two sampling stations were located downstream of the injection point, one at the center of the stream and another at a distance of seven feet from the right bank. Time-concentration samples were collected at these points which were located 200 feet downstream of the tracer injection

point for the first test and 400 feet downstream for the second test. Samples were collected in 100 ml glass bottles (bacterial dilution bottles) by moving the bottles rapidly up and down as they filled. The intent of this collection method was to obtain a vertically integrated sample.

The river was gaged by standard discharge measurement procedures (39) for the flows encountered. The stage height was measured by steel tape and plumb-bob from a reference point located on top of a concrete railing post on the upstream side of the State Highway 10 bridge just downstream of the test reach. Although the U. S. Geological Survey maintains a gaging station less than two river miles upstream of the test reach, the data available there was deemed unacceptable due mainly to an impoundment located between the two sites.

The procedure for each field test consists of the following steps:

1. A measured mass of powdered Rhodamine B (100 to 120 grams) is dissolved in a 40% by weight solution of acetic acid.

2. Standards are prepared from the above stock solution using river water for make-up.

3. All sample bottles to be used in the test are washed and labeled.

4. The required apparatus are brought to the test reach and the river stage is recorded.

5. The desired sampling points and the tracer injection points are identified by measuring the appropriate distances with a 100 foot steel tape. Each point is marked by driving five foot steel poles into the river bottom.

6. The tracer is injected.

7. Vertically integrated samples are collected at the specified times. A plastic bucket is floated near the observation point to store the sample bottles during collection.

8. The samples and standards are removed directly to the laboratory for analysis. Standard solutions are analyzed and calibration curves prepared.

9. The samples are analyzed and the standards are re-run to determine whether significant drift has occurred in the instrument. (If drift is significant the samples must be analyzed again.)

10. The sample concentrations are determined from the calibration curves.

11. The data is read into the computer program and the resulting coefficients are determined.

5.2. Results

The results of the two field tests are reported in Tables I and II. The stream width reported is that of the approximate mean width of the trapezoidal channel. The stream depth is given as the average depth encountered in the cross-section. The stream velocity reported was determined by time of travel for the peak concentration recorded at the mid-stream sampling point. The data were analyzed by both the Gradient and the Newton-Raphson methods as outlined in Section 4.4.2. The FORTRAN IV programs utilized are presented in Appendix A.

Table I. Results of Field Test Number One

		Station A		Station B	
Time (sec)	x (feet)	v (feet)	c (ppb)	y (feet)	c (ppb)
60	200	0	0	15	0
120	200	0	1250	15	0
150	200	0	1600	15	220
180	200	0	90	15	0
210	200	0	20	15	0
240	200	0	10	15	0
270	200	0	10	15	0
300	200	0	0	15	0

Date: May 27, 1970

Location: Mill River at Northampton, 500 ft above State Highway 10 bridge.

Variables:

Avg. stream velocity, U = 1.3 ft/sec

Mass of dye injected, M = 200 grams

Width of stream, W = 44 ft

Lateral injection point, w = 22 ft

Average stream depth, d = 3.3 ft

Results:

Longitudinal dispersion coefficient, E = 5.2 ft^2/sec

Lateral diffusion coefficient, D_y = 0.5 ft^2/sec

Figure 16. Results of Field Test 1.

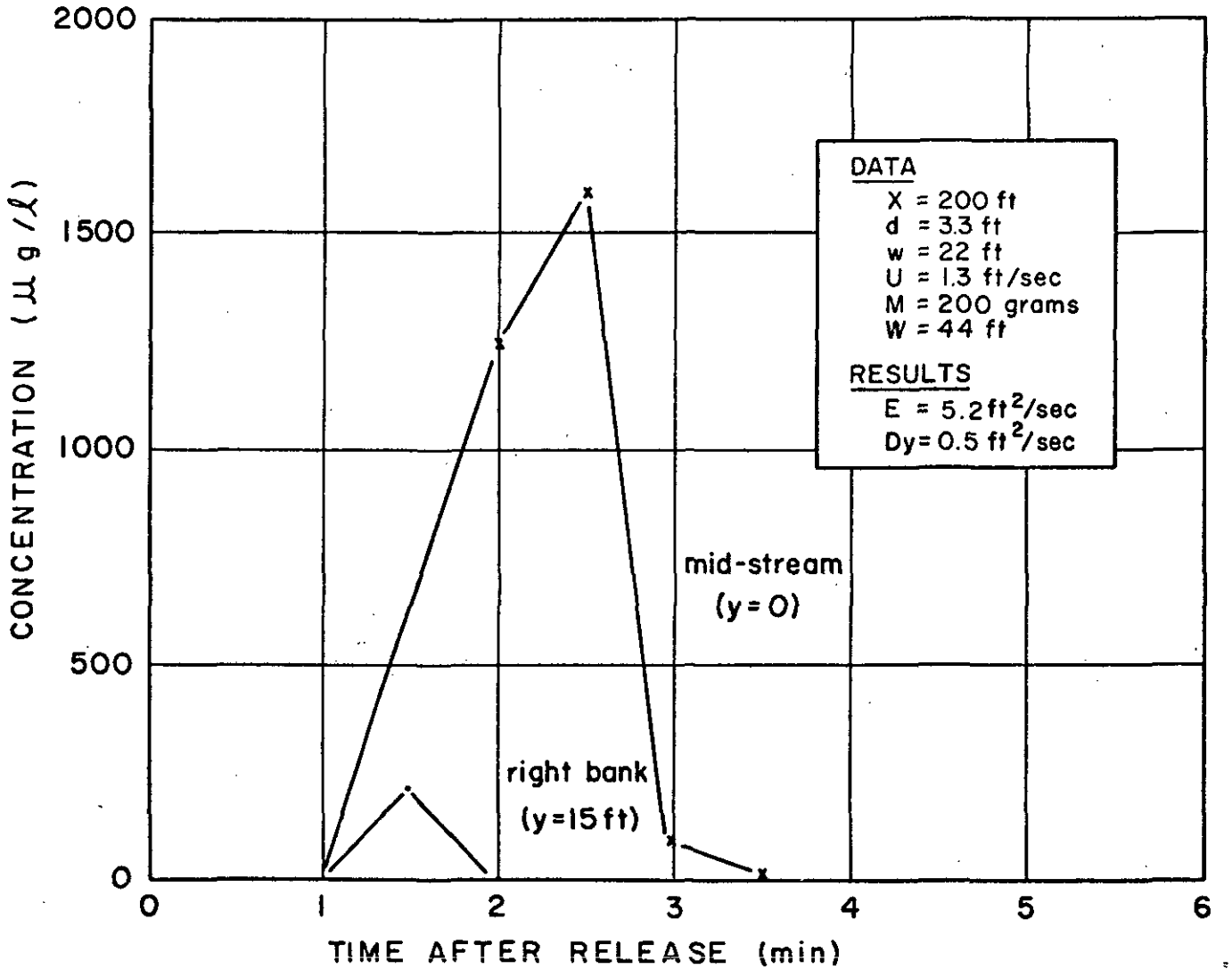


Table II. Results of Field Test Number Two

Time (sec)	x (feet)	Station A y (feet)	Station A c (ppb)	Station B y (feet)	Station B c (ppb)
180	400	0	0	15	0
210	400	0	0	15	0
240	400	0	12	15	0
255	400	0	10	15	0
270	400	0	450	15	0
285	400	0	920	15	270
300	400	0	850	15	185
315	400	0	440	15	50
330	400	0	280	15	160
345	400	0	170	15	210
360	400	0	95	15	275
390	400	0	30	15	180
420	400	0	0	15	100
480	400	0	0	15	30
540	400	0	0	15	0
600	400	0	0	15	12
720	400	0	0	15	0

Date: June 4, 1970

Location: Mill River at Northampton, 500 ft above State Highway 10 bridge

Variables:

Avg. stream velocity, V = 1.4 ft/sec

Mass of dye injected, M = 112 grams

Width of stream, W = 44 ft

Lateral injection point, w = 22 ft

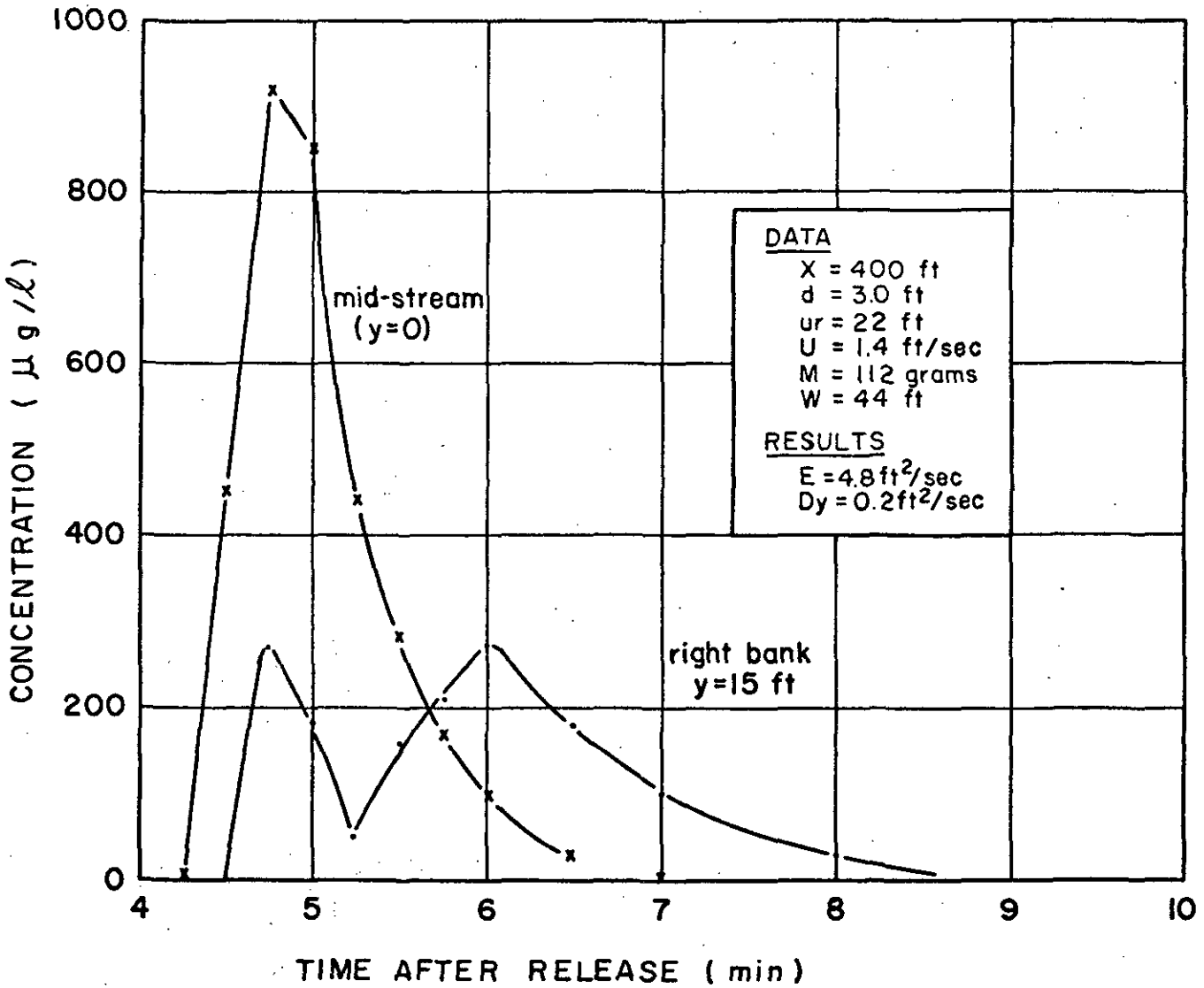
Average stream depth, d = 3.0 ft

Results:

Longitudinal dispersion coefficient, E = 4.8 ft^2/sec

Lateral diffusion coefficient, D_y = 0.2 ft^2/sec

Figure 17. Results of Field Test 2.



5.3. Discussion

The values obtained for the lateral diffusion and the longitudinal dispersion coefficients are considerably smaller than values reported in the literature for estuaries; however, they closely approximate the values suggested by Holley (8) for streams. In observing the dye mass slug as it traveled downstream one could not help remark how slowly the mass did spread. The dye mass seemed to move at least 200 feet before reaching the lateral boundaries. When the mass did reach the lateral boundaries, the dye appeared to accumulate near the boundaries. This caused the dye mass to form into a horseshoe pattern, as the mass continued to mix; however, the horseshoe pattern evolved more into an elliptical pattern with truncation at the stream banks.

The methods used for the dye dispersion studies were quite straight forward and proceeded smoothly from start to finish.

The results of the experimental tests indicate that the dispersion and diffusion coefficients for streams are much lower than the values frequently quoted for estuaries. Nevertheless, the results obtained here are considered to be consistent with the mixing theory incorporated within this paper. The experimental procedure for determining values for D_y and E by the image method is far less consumptive of time and resources than the other techniques described. As a result there are fewer problems which might arise during the test. Other procedures require excessive manpower and materials (gallons of dye are often needed). The complications are many and often casual oversights or mistakes can cause the whole survey to fail, bringing on unnecessary and rather significant expense.

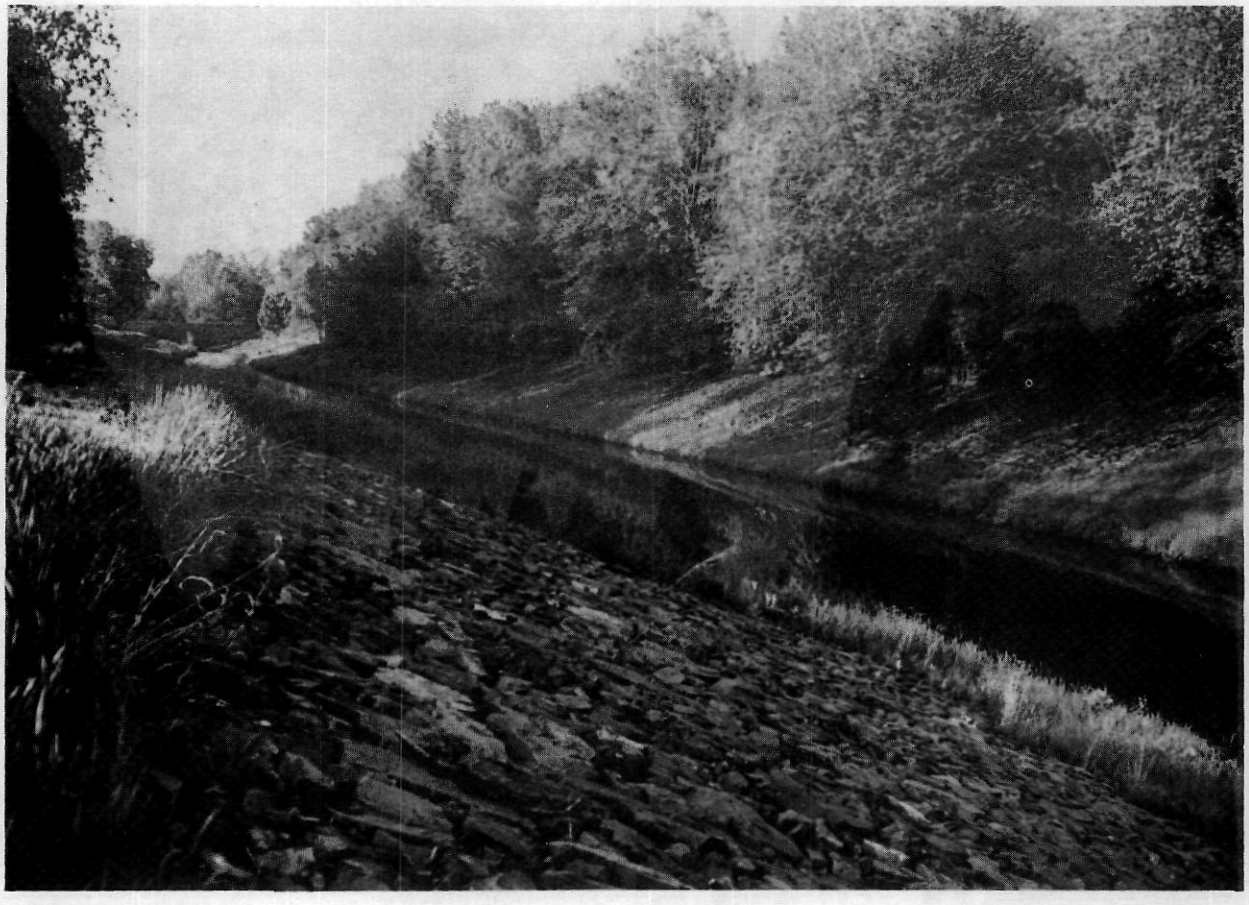


FIGURE 18. Photograph of Test Stream - Mill River at Northampton above State Highway 10 Bridge.

Although the use of a high speed digital computer is prerequisite to the use of the method presented in this paper, the cost of this facility is minimal when compared with the cost of manpower needed for the other methods.

CHAPTER 6 SUMMARY AND CONCLUSIONS

In the past decade significant effort has been devoted toward developing mathematical models of stream and estuary water quality. Such models are fundamental to economic wastewater treatment plant design in order to meet water quality standards. A water quality model must account for all significant biological, chemical and physical mechanisms which affect the distribution of the parameter being modeled. This paper has dealt only with the physical processes affecting pollutant transport. The physical model normally serves as the foundation upon which the reactive properties of a particular parameter are superimposed. The physical aspects must then be well formulated if the model is to be successful.

The concept of plug flow is completely unacceptable for estuarine systems and is often unrealistic for streams. In turbulent stream flow mixing of dissolved particles occurs by molecular diffusion, turbulent diffusion and by dispersion. The term "diffusion" is applied to random motion. Molecular diffusion is an extremely slow mixing process caused by Brownian molecular motion. Turbulent diffusion is caused by random time-velocity fluctuations and thus causes equal but somewhat inefficient mixing in all directions. Dispersion, the most efficient mixing process, is primarily a fixed process occurring as a direct result of the spatial velocity distribution in a stream. Mathematically, dispersion incorporates longitudinal diffusion.

Mixing zones are established downstream of a point outfall. The three-dimensional mixing zone extends downstream from an outfall to a point where all vertical concentration gradients disappear. This zone

or reach is typically very short and for practical purposes may be neglected in most stream models. In the two-dimensional mixing zone concentration gradients exist in the longitudinal and lateral directions only. A second order partial differential equation describing mass transport in this zone has been derived, but due to the finite extent of the lateral stream boundaries, an analytic solution has not been found. However, for an infinite flow field the equation is readily solved. It has been proposed herein that the physical boundaries can be replaced with an equivalent hydraulic system utilizing classical image theory, also known as the principle of reflection. The solution to such a system is obtained by the superposition of an infinite series of image outfalls. For practical purposes, however, only a small number of imaginary outfalls need be considered as the effect of image sources diminishes with distance away from the real system.

The equation governing one-dimensional mixing and advection is readily solved analytically. Since coefficients of dispersion cannot be measured directly,the one-dimensional transport solution is utilized in conjunction with tracer injections in order to calculate values for the dispersion coefficient. Various methods are available to accomplish this, however all are expensive and time consuming in their applications. Furthermore, the result is a single value which must then serve as the average for several hydraulically heterogeneous river miles. Specific values for short and relatively homogeneous reaches cannot be obtained without costly and tedious survey methods.

One of the most prominent applications of the solution to the two-dimensional mixing equation proposed here is a new, relatively simple and much less costly method of obtaining longitudinal dispersion coefficient values. Furthermore, the method allows one to obtain specific values of the lateral diffusion coefficient for short reaches. Very little previous work has been aimed at evaluating this coefficient in natural streams.

The method proposed of evaluating E and D_y was utilized for a local stream. The results obtained suggest that coefficients of dispersion in natural streams are probably far smaller than those often reported in contemporary literature.

The series solution presented for the two-dimensional mixing equation requires excessive mathematical operations in predicting spatial and temporal concentration distributions. Such calculations are, however, well suited to electronic digital computation. A useful FORTRAN IV program has been included in the Appendix for applying the image solution to obtain values for dispersion and lateral diffusion coefficients from experimental field data utilizing the procedures outlined in Section 4.4.2 of this paper.

Aside from its application in turbulent mixing studies, the two-dimensional transport solution will find relevance in more accurately predicting the effect of nearby outfalls on industrial and water supply intakes, bathing beaches, and on the natural stream ecology. The two-dimensional solution allows direct prediction of any conservative

pollutant concentration in the lateral and longitudinal plane of a stream due to a nearby outfall. The reactive properties of non-conservative pollutants may be applied to the transport solution to obtain similar concentration predictions.

The three-dimensional mixing-advection equation may also be solved by the image method; however, for relatively shallow streams vertical mixing occurs quite rapidly and the use of three-dimensional mixing is of little value. For deep, slowly moving streams, the three-dimensional solution will be useful in predicting pollutant concentrations at any lateral, vertical and longitudinal point due to nearby wastewater outfalls. This suggests application of the method to estuarine systems. In deep streams and relatively deep estuaries, density stratification is common. The image solution might find useful application in such problems where an exchange mechanism operating between the density zones could be mathematically incorporated into the three-dimensional image solution.

In conclusion, it is hoped that the practical solution given to the natural stream mixing equation will be evaluated for its merit by contemporary investigators. With the solution given, specific concentration predictions at any location near outfalls should allow for more durable and efficient wastewater treatment plant design and outfall location. Finally, the method given for determining E presents a less costly, localized and more direct procedure for evaluating this elusive parameter.

APPENDIX

A.1 FORTRAN IV PROGRAM UTILIZING GRADIENT METHOD TO DETERMINE LONGITUDINAL DISPERSION AND LATERAL DIFFUSION COEFFICIENTS FROM FIELD TEST DATA.

```
10 PROGRAM DAVE4
20 DIMENSION C(50),T(50),Y(50),EQ(50),AE(5,5),D(5,5),SSD(5,5)
25 M1=34
30 READ,(C(MMM),MMM=1,M1)
40 READ,(T(MMM),MMM=1,M1)
50 READ,(Y(MMM),MMM=1,M1)
51 DO 55 MMM=1,M1
52 T(MMM)=T(MMM)*60.0
55 CONTINUE
60 X=400.0
70 AM=112.0*10.0**6.0/28.31
80 U=1.4
90 SW=44.0
100 D1=3.0
110 OW=22.0
120 SAE=4.763004590 $ SD=0.2347307237
130 SX=0.1
135 DO 225 MM=1,3
136 DO 225 NN=1,3
137 ANNN=NN
138 AMM=MM
140 AE(MM,NN)=SAE-(SX*(AMM-2.0))
145 D(MM,NN)=SD+(SX*(ANNN-2.0))
150 SSD(MM,NN)=0.0
155 DO 220 MMM=1,M1
156 EQ(MMM)=0.0
160 DO 205 M=0,5
170 DO 205 N=1,4
180 A=AM/(D1*4.0*3.1416*T(MMM))
190 AA=-(1.0/(4.0*T(MMM)))*(X-U*T(MMM))**2
195 AAAM=M
196 ANN=(-1)**N+1
197 AAAN=N
200 AAA=(-0.25/(T(MMM)))*(Y(MMM)+2.0*AAAM*SW*(2.0)**0.5*SINF(
201C(2.0*AAAN-1.0)*3.1416/4.0)-OW*ANN)**2
202 EQ(MMM)=EQ(MMM)+(A/(AE(MM,NN)*D(MM,NN))**0.5*EXPF(AA/A
203CE(MM,NN)+AAA/D(MM,NN)))
205 CONTINUE
210 SSD(MM,NN)=SSD(MM,NN)+(C(MMM)-EQ(MMM))**2
220 CONTINUE
225 CONTINUE
230 MS=0
240 =SSD(2,2)
245 PRINT,((SSD(MM,NN),NN=1,3),MM=1,3)
```

```
250 DO 290 MM=1,3
260 DO 290 NN=1,3
270 IF (S-SSD(MM,NN))290,290,280
280 S=SSD(MM,NN)
281 SAE=AE(MM,NN) $ SD=D(MM,NN)
285 MS=MS+1
290 CONTINUE
310 IF (MS)350,350,320
315 PRINT,S
320 GO T 130
350 PRINT,SAE,SD
355 END
360 ENDPROG
```

DATA INPUT

A.2 FORTRAN IV PROGRAM UTILIZING NEWTON-RAPHSON METHOD TO DETERMINE LONGITUDINAL DISPERSION AND LATERAL DIFFUSION COEFFICIENTS FROM FIELD TEST DATA.

```
10 PROGRAM DAVF3
20 DIMENSION C(50),T(50),Y(50)
25 M1=8
30 READ,(C(MMM),MMM=1,M1)
40 READ,(T(MMM),MMM=1,M1)
50 READ,(Y(MMM),MMM=1,M1)
51 DO 55 MMM=1,M1
52 T(MMM)=T(MMM)*60.0
55 CONTINUE
60 X=200.0
70 AM=200.0*10.0**6.0/28.31
80 U=1.3
90 SW=44.0
100 D1=3.3
110 OW=22.0
120 AE=5.029 $ D=0.542
123 SUMF1=0.0 $ SUMF2=0.0 $ SUMF3=0.0 $ SUMH1=0.0 $ SUMH2=0.0
124 SUMH3=0.0
125 DO 490 MMM=1,M1
130 SUM1=0.0
131 SUM2=0.0
132 SUM5=0.0
133 SUM6=0.0
134 SUM7=0.0
135 SUM8=0.0
136 SUM9=0.0
137 SUM10=0.0
138 SUM11=0.0
139 SUM12=0.0
140 SUM13=0.0
142 SUM15=0.0
143 SUM16=0.0
144 SUM17=0.0
145 SUM18=0.0
146 SUM19=0.0
148 SUM21=0.0
149 SUM23=0.0
160 DO 420 M=0,5
170 DO 420 N=1,4
180 A=AM/(D1*4.0*3.1416*T(MMM))
190 AA=-(1.0/(4.0*T(MMM)))*(X-U*T(MMM))**2
195 AAAM=M
196 ANN=(-1)**N+1
197 AAAN=N
200 AAA=(-0.25/(T(MMM)))*(Y(MMM)+2.0*AAAM*SW*(2.0)**0.5*SINF(
201C(2.0*AAAN-1.0)*3.1416/4.0)-OW*ANN)**2
210 R=EXPF(AA/AE+AAA/D)
220 SUM1=SUM1+R*(A*AA/(AE**2.5*D**.5)+0.5*A/(AE**1.5*D**.5))
230 SUM2=SUM2+R*(A/(AE**0.5*D**0.5))
```

```
240 SUM5=SUM5-B*(0.5*A*AA/(AE**3.5*D**0.5)+0.75*A/(AE**2.5
241C*D**0.5))
250 SUM6=SUM6-B*(A*AA**2/(AE**4.5*D**0.5)+2.5*A*AA/
251C(AE**3.5*D**0.5))
260 SUM7=SUM7-B*(A*AA/(AE**2.5*D**0.5)+0.5*A/(AE**1.5*
261CD**0.5))
270 SUM8=SUM8-B*(0.5*A*AAA/(AE**1.5*D**2.5)+0.25*A/(AE**1.5
271C*D**1.5))
280 SM9=SUM9-B*(A*AA*AAA/(AE**2.5*D*2.5)+0.5*A*AA/(AE**2.5*
281CD**1.5))
290 SUM10=SUM10-B*(0.5*A*AAA/(AE**1.5*D**2.5)+0.25*A/(AE**1.5*
291CD**1.5))
300 SUM11=SUM11-B*(A*AAA/(AE**0.5*D**2.5)+0.5*A/(AE
301C**0.5*D**1.5))
310 SUM12=SUM12-B*(A*AA*AAA/(AE*2.5*D**2.5)+0.5*A*AAA/(AE**1.5*
311CD**2.5))
320 SUM13=SUM13-B*(0.5*A*AA/(AE**2.5*D**1.5)+0.25*A/(AE**1.5*
321CD**1.5))
340 SUM15=SUM15-B*(0.5*A*AA/(AE**2.5*D**1.5)+0.25*A/(AE
341C**1.5*D**1.5))
350 SUM16+B*(A*AAA/(AE**0.5*D**2.5)+0.5*A/(A**0.5*D**1.5))
360 SUM17=SUM17-B*(A*AA/(AE**2.5*D**0.5)+0.5*A/(AE**1.5*D**0.5))
370 SUM18=SUM18-B*(A*AAA**2/(AE**0.5*D**4.5)+2.5*A*AAA
371C/(AE**0.5*D**3.5))
380 SUM19=SUM19-B*(0.5*A*AAA/(AE**0.5*D**3.5)+0.75*A
381C/(AE**0.5*D**2.5))
395 SUM21=SUM2-B*(0.5*A*AAA/(AE**0.5*D**3.5)+0.75*A/(AE**0.5*
396CD**2.5))
400 SUM23=SUM23-B*(A*AAA/(AE**0.5*D**2.5)+0.5*A/(
401CAE**0.5*D**1.5))
420 CONTINUE
430 SUMF1=SUMF1+C(MMM)*SUM1-SUM2*SUM1
440 SUMH1=SUMH1+C(MMM)*SUM16-SUM2*SUM16
450 SUMF2=SUMF2+CMMM)*(SUM5+SUM6)-(SUM2*(SUM5+SUM6)+SUM1*S
451CUM7)
460 SUMF3=SUMF3+C(MMM)*(SUM8+SUM9)-(SUM2*(SUM10+SUM9)+SUM1*
461CSUM11)
470 SUMH2=SUMH2+C(MMM)*(SUM12+SUM13)-(SUM2*(SUM12+SUM15)+SUM1
471C6*SUM17)
480 SUMH3=SUMH3+C(MMM)*(SUM18+SUM19)-(SUM2*(SUM18+SUM1)+SU
481CM16*SUM23)
490 CONTINUE
495 PRINT,SUMF1,SUMF2,SUMF3,SUMH1,SUMH2,SUMH3
500 DET=SUMF2*SUMH3-SUMF3*SUMH2
510 ALPHA=AE-1.0/DET*(SUMF1*SUMH3-SUMF3*SUMH1)
520 BETA=D-1.0/DET*(SUMF2*SUMH1-SUMF1*SUMH2)
525 PRINT,ALPHA,BETA
530 IF (ABSF(LPHA-AE)-0.00001)560,560,540
540 AE=ALPHA $ D=BETA
550 GO TO 123
560 END
570 ENDPROG
```

DATA INPUT

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